Part A: Questions 1-6

Each question should be answered by a single choice from A to E.
每题有五个选项，考生应从中选出一个正确答案。
Questions are worth 3 marks each.
每题 **3** 分。

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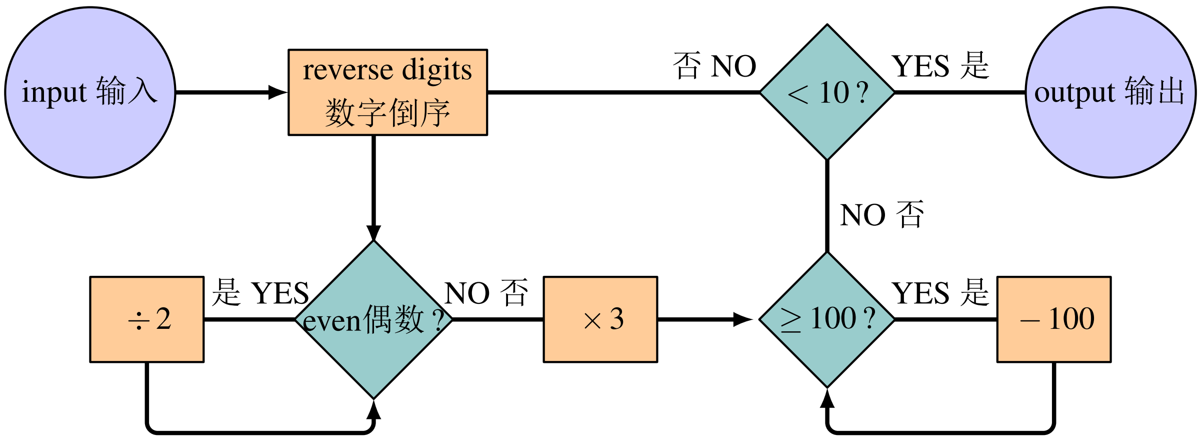
1. Flow-流程图

The number 38 is input into the following flow chart:

将数字 38 输入下列流程图：



How many times is the ‘reverse digits’ process applied before ‘output’ is reached?

请问在“输出”之前数字 38 要经过多少次“数字倒序”？

(A) 1 (B) 2 (C) 3

(D) 4 (E) ‘output’ is never reached 永远无法“输出”

2. Growth-野蛮生长

The game of *Growth* takes place on a grid of white squares.

野蛮生长 游戏采用一个由白色方格组成的网格。

- To start, a single square somewhere in the grid is chosen. It is shaded.

  游戏开始时，网格中随机出现一个阴影方格。

- To make a move, a player chooses a shaded square and shades two white squares that are adjacent to it. (Adjacent means the squares have an edge in common.)

  玩家每一轮需要选择一个阴影方格，然后为其两个相邻白色方格涂上阴影。(相邻方格指的是共用一条边的方格。)

- A shaded square **cannot** be chosen if it does not have two white squares adjacent to it.

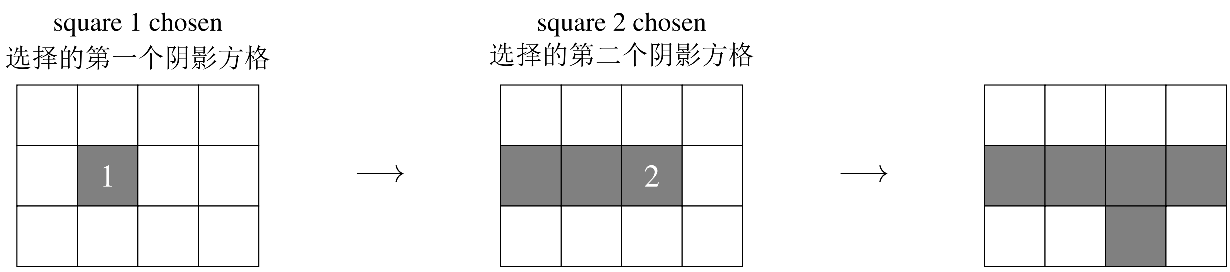
  如果一个阴影方格没有两个相邻的白色方格，那么它就**不能**被选择。

- A shaded square **can** be chosen more than once.

  一个阴影方格**可以**被多次选择。

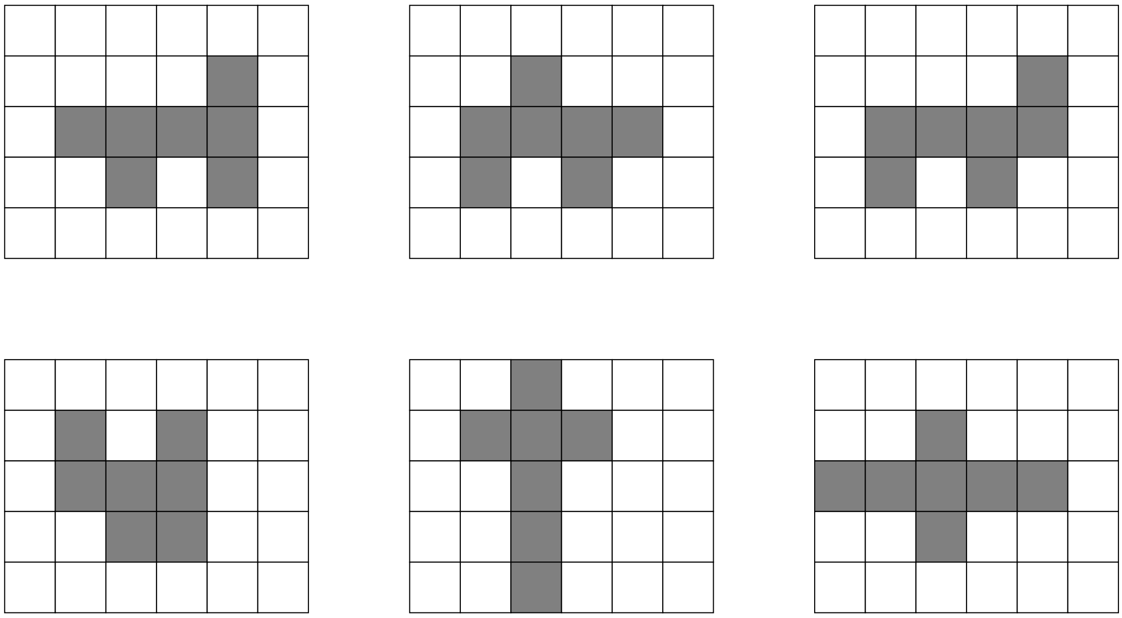
The example below shows the first shaded square and how the board *could* develop over two moves on a 4×3 grid.

下图显示了在这个 4×3 网格中选择的第一个阴影方格以及经过两轮操作后网格可能的变化情况。



How many of the following diagrams could represent the game position after three moves?

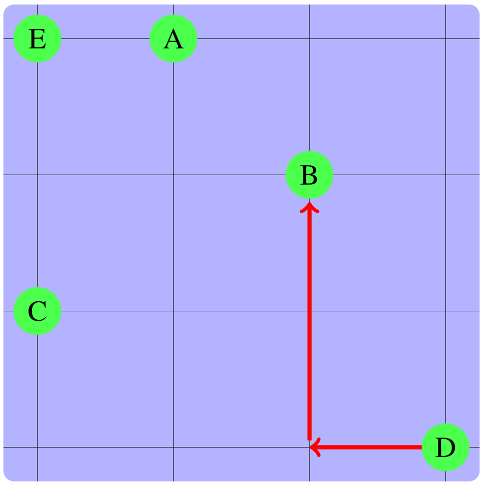
请问在下列图片中，有多少张图片显示的是三轮操作之后的结果？



(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

3. Magnetic drone-磁力飞侠

Magnetic drones can only fly along magnetic field lines or perpendicular to them. Their instructions are of the form $m \rightarrow \; n \downarrow$.
磁吸无人机只能沿着磁场边线或垂直于这些边线飞行。该无人机指令形式为 $m \rightarrow \; n \downarrow$。



For instance, the instruction $1 \leftarrow \; 2 \uparrow$ could be used to fly from island D to island B.
例如，无人机执行指令 $1 \leftarrow \; 2 \uparrow$，可从 D 岛飞往 B 岛。

David's drone started on one of the islands, but we don't know which one. It then flew to three other islands and landed on the fourth. Thus it spent time on all of the islands exactly once, and did not return to its starting point.

David 的无人机从其中某个岛屿起飞，经过其他三个岛屿后，在第四个岛屿降落。此次飞行中，这架无人机对所有岛屿都恰好进行了一次飞越，并且它并未返回起飞岛屿。

To carry out this tour, it used exactly four of the instructions below in some order.

此次飞行，无人机恰好按某种顺序执行了以下指令中的四种指令。

$2 \rightarrow \; 2 \uparrow$

$2 \leftarrow \; 3 \uparrow$

$2 \rightarrow \; 1 \downarrow$

$2 \leftarrow \; 1 \uparrow$

$1 \rightarrow \; 2 \downarrow$

$1 \leftarrow \; 2 \downarrow$

On which island did the drone start?

请问无人机是从哪个岛屿起飞的？

(A) A (B) B (C) C (D) D (E) E

4. Avalon-阿瓦隆

You have to be careful when travelling in Avalon – there are leprechaun woods and faerie glens.

在阿瓦隆旅游时切记注意安全——阿瓦隆里有一片神秘的矮妖森林和精灵幽谷。

- When you pass through a leprechaun wood you must wear green.

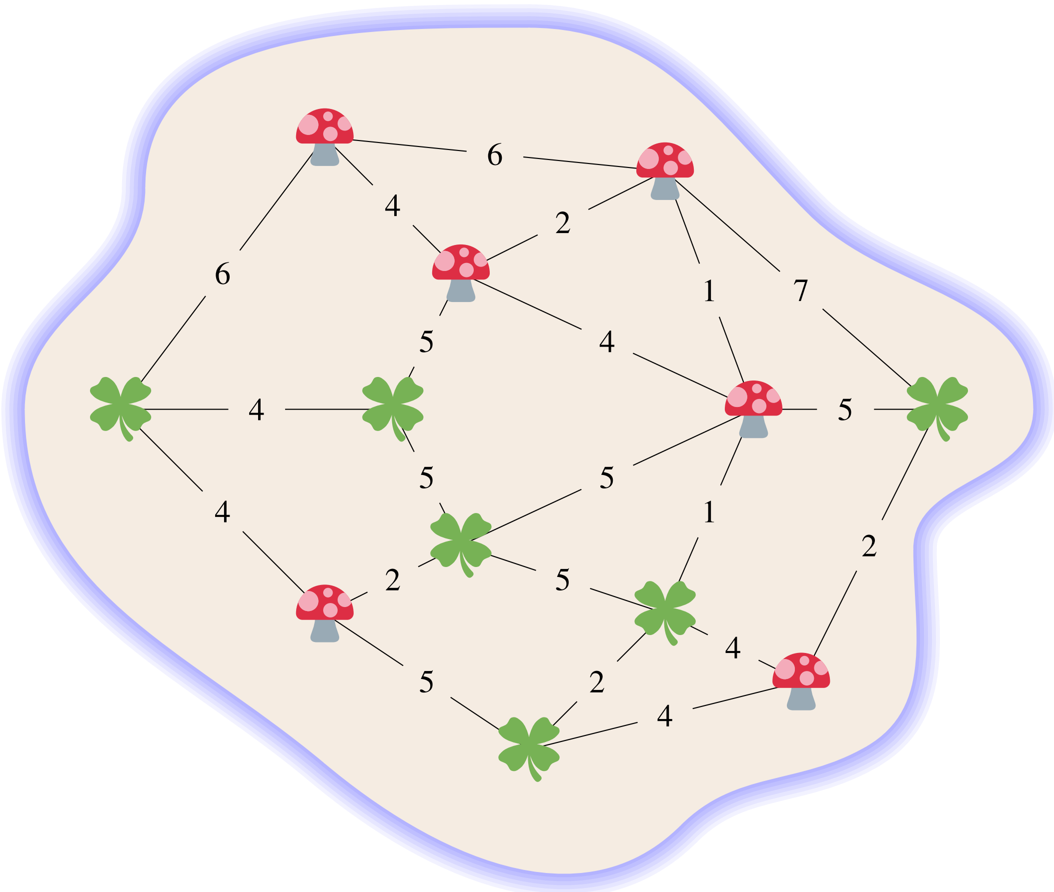
  当你穿过矮妖森林时，必须穿绿色衣服。

- When you pass through a faerie glen you must wear pink.

  当你穿过精灵幽谷时，必须穿粉色衣服。

In the map below, the woods are indicated by  and glens by . The numbers represent your travel time in hours.

下图中，矮妖森林用表示；精灵幽谷用表示。其中数字代表你的穿行时长，以小时为单位。



You wish to make a journey in the shortest time possible.

假设你希望用尽可能短的时间完成在阿瓦隆的旅行。

- You start in the leftmost wood, wearing green.

  旅行开始时，你穿着绿色衣服，位于最左侧的森林中。

- You finish in the rightmost wood, again wearing green.

  旅行结束时，你再次穿着绿色衣服，位于最右侧的森林中。

- You have a green outfit and a pink outfit.
  你有一套绿色衣服和一套红色衣服。

- It takes you 2 hours to unpack, change outfit and repack.
  整趟旅行中，“拿出衣服”、“换上衣服”，再“收整衣服”这三个动作加起来一共花费 2 小时。

What is the shortest possible time, in hours, for you to finish your journey?

请问完成整趟旅行最少可能需要几个小时？

(A) 17 (B) 19 (C) 21 (D) 23 (E) 25

5. Row delete–消消乐

In a spreadsheet, rows are labelled 1, 2, 3, ... and columns are labelled A, B, C, ...

在下列图表中，行标记为 1、2、3、...；列标记为 A、B、C、...

When a row is deleted, all of the rows below it shuffle up one position. For example, deleting row 3 means the *new* row 3 is the *old* row 4, and so on.

每消除一行，此行下方的所有行上移一行。例如，消除第 3 行时，原来的第 4 行就会成为新的第 3 行，以此类推。

	A	B	C
1	Antares	Scorpio	M
2	Rigel	Orion	B
3	Pollux	Gemini	K
4	Canopus	Carina	A
5	Acrux	Crux	B

delete 消除 → row 3 第 3 行

	A	B	C
1	Antares	Scorpio	M
2	Rigel	Orion	B
3	Canopus	Carina	A
4	Acrux	Crux	B
5	Polaris	Ursa Min	F

Rows are deleted one at a time by a sequence of row numbers such as 3, 5, 2, ...

给定一个行号序列，例如 3、5、2、...，按序列中的顺序逐一消除各行。

This means delete row 3, then delete the *new* row 5 (which is the original row 6), then delete the *new* row 2 (which is the original row 2), and so on.

首先消除第 3 行，再消除新的第 5 行（即原先的第 6 行），之后消除新的第 2 行（即原先的第 2 行），以此类推。

Different sequences may or may not have different effects. For example, deleting $3,1,3$ has the same effect as deleting $5,1,2$. Both of these delete, in some order, the rows that were originally numbered 1, 3 and 5.

不同序列可能会也可能不会产生不同效果。例如，按照 $3,1,3$ 的顺序消除行的效果与按 $5,1,2$ 的顺序消除行的效果相同。以上两种都按某种顺序消除了原先的 1、3、5 行。

The cost of a sequence is the sum of the row numbers: $3,1,3$ has cost $3+1+3=7$ and $5,1,2$ has cost $5+1+2=8$. So $3,1,3$ is cheaper than $5,1,2$.

序列成本即序列中行号总和，例如序列 $3,1,3$ 的成本为 $3+1+3=7$；序列 $5,1,2$ 的成本为 $5+1+2=8$。因此， $3,1,3$ 的成本比 $5,1,2$ 的成本更低。

What is the cost of the cheapest sequence that has the same effect as $3,1,4,1,5,9,2,6,5$?

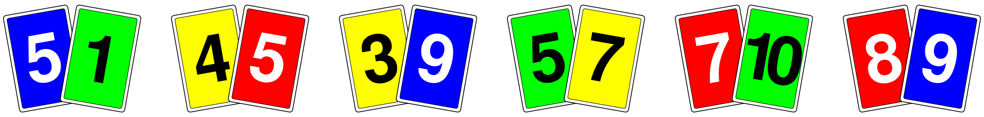
请问与序列 $3,1,4,1,5,9,2,6,5$ 效果相同的序列中，最便宜序列的成本是多少？

(A) 21 (B) 23 (C) 25 (D) 27 (E) 29

6. Card choices-抽卡

Tyson is playing a card game. He has six pairs of numbered cards, all visible, as follows:

Tyson 正在玩一种卡牌游戏。他有六组可见卡牌，如图所示：



Tyson can choose *at most one* card from each pair, and aims to make the highest possible total.

Tyson 从每组卡牌中最多抽取一张卡牌，使得抽取的卡牌总和尽可能达到最大值。

However, as he moves from left to right, any card he chooses must be higher than the card previously chosen.

但是，当他从左向右抽取卡牌时，他抽取的任何一张卡牌上的数字必须比前一张抽取的卡牌上的数字大。

Examples 举例

Valid sequence 有效序列: 5 _ 9 _ 10 _

Invalid sequence 无效序列: 1 5 3 _ _ _

What is the highest total Tyson can make?

Tyson 抽取的卡牌上的数字总和最大能达到多少？

(A) 24 (B) 25 (C) 26 (D) 27 (E) 28

Part B: Questions 7–9

Each question has three parts, each of which is worth 2 marks.
每题有三个部分，每部分 2 分。
Each part should be answered by a number in the range 0–999.
每部分答案应为一个介于 0–999 之间的数字。

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7. Multiswap-移形换影

You have a line of ✓s and ✗s. You want all of the ✓s to be on the left and the ✗s to be on the right.

你有一个由多个 ✓和多个 ✗ 组成的行。现在你想要把所有的 ✓放在左边，所有的 ✗ 放在右边。

You will do this by several rounds of swapping. In each round:

该目标可以通过多轮交换完成。每一轮：

1. a move consists of swapping a ✓with the ✗ immediately before it

   一次交换指的是将✓与紧邻 ✓的前一个 ✗ 交换

2. there can be several moves in a round

   每一轮可以进行多次交换

3. neither a ✓ nor a ✗ can be part of more than one move in a round.

   每一个 ✓ 和 ✗ 都不能在一轮中参与多次交换。

Example 举例:

✗	✗	✓	✗	✓	→	✗	✓	✗	✗	✓	Round with one swap 仅一次交换的轮次
✗	✗	✓	✗	✓	→	✗	✗	✓	✓	✗	Round with one swap 仅一次交换的轮次
✗	✗	✓	✗	✓	→	✗	✓	✗	✓	✗	Round with two swaps 经历两次交换的轮次

For each of the following lines, what is the fewest number of rounds to move all of the ✓s to the left end of the line?

对于下列各行，如果要将所有的 ✓放在行的左边，最少要经过几轮交换？

A. ✗ ✗ ✗ ✓ ✓ ✓ ✓ ✓

B. ✗ ✗ ✓ ✗ ✗ ✗ ✗ ✓ ✓ ✓ ✓

C. ✗ ✗ ✓ ✓ ✓ ✓ ✗ ✗ ✓ ✓ ✓ ✗ ✗ ✓

8. Bus passengers-公交乘客

A bus is travelling from A to B, with several stops along the way.

一辆公交车从 A 点开往 B 点，沿途有一些经停站点。

At each stop several passengers get on and get off, as shown in the following tables.

每到达一个经停点时，都有一些乘客上下车，如下列表所示。

Every passenger travels at least one leg. No-one gets on and off at the same stop.

每位乘客至少搭乘一站，且没有人在同一站点上下车。

For each of the following trips, what is the greatest number of passengers who could have travelled from A to B for the entire journey?

对于下列每种公交线路，计算可以从 A 点一直搭乘到 B 点下车的最大乘客数量。

A.

Stop 经停站点	A	1	2	3	4	5	6	B
Passengers on 上车人数	20	10	0	5	0	5	0	–
Passengers off 下车人数	–	0	8	0	10	0	3	19

B.

Stop 经停站点	A	1	2	3	4	5	B
Passengers on 上车人数	20	10	8	4	4	2	–
Passengers off 下车人数	–	0	8	6	8	4	22

C.

Stop 经停站点	A	1	2	3	4	5	6	B
Passengers on 上车人数	20	5	6	4	8	5	5	–
Passengers off 下车人数	–	8	3	6	10	7	7	12

9. Capri 山羊

You are using your goats to clear blackberry thickets. Your goats are contrary creatures and can't be told what to do.

你正在驱赶山羊去清洁黑莓灌木丛，但这些山羊都很叛逆，并不会完全听从你的指令。

1. Goats will only work for whole days.

   山羊的工作时间只能按整天计算。

2. If a goat has been assigned to a thicket, it will not permit any other goat, except Capri, help it clear the thicket.

   如果某只山羊被分配到某个灌木丛，那么这只山羊不会允许除 Capri 之外的其他山羊来帮它清理这个灌木丛。

3. Each goat can clear an area of 1 GoatRood (GR) of blackberries per day.

   每只山羊每天只能清理 1 GR 的黑莓。

4. Capri:

   i. Capri will not work by himself.
   Capri 不会单独工作。

   ii. The other goats will let Capri join them.
   其他山羊会邀请 Capri 一起工作。

   iii. If Capri helps another goat, they clear 2 GR of blackberries per day.
   如果 Capri 帮助其他山羊一起清洁，那么这两只山羊一天就能清洁 2 GR 的黑莓。

Each of the other goats has been assigned a blackberry thicket. You know the area of each thicket, and therefore how many days one goat would take to clear it. You want to use Capri so that all thickets are cleared in as few days as possible (6 days and 12 days in the examples below).

除 Capri 之外的每一只山羊都被分配了一片黑莓灌木丛。你了解每片灌木丛的面积，所以可以计算出每只山羊要花多少天的时间才能清理完它们被分配到的灌木丛。现在你想派出 Capri 参加清洁任务，这样一来就可以将所有灌木丛的总清洁时间尽可能地缩短。（下表展示两个分别需要 6 天和 12 天才能清洁完毕的案例）

Examples (Rem. is the area remaining to be cleared.)
举例 (Rem.指的是剩余待清洁区域.)

Two thickets, of areas 11 and 4 GR.
两片灌木丛，面积分别为 11 和 4 GR。
Capri (*) helps clear thicket 1 for the first 5 days.
Capri(*) 在前 5 天协助清洁 1 号灌木丛。

English version 英文版本:

	Thicket 1 (11)		Thicket 2 (4)	
Days	Cleared	Rem.	Cleared	Rem.
1–4	8*	3	4	0
5	2*	1		
6	1	0		

Chinese version 中文版本：

	1 号灌木丛(11)		2 号灌木丛(4)	
天数	已清洁	待清洁.	已清洁	待清洁.
1–4	8*	3	4	0
5	2*	1		
6	1	0		

6 days are required to clear the thickets.
所有灌木丛清洁完毕需要 6 天。

Two thickets, of areas 20 and 15 GR.
两片灌木丛，面积分别为 20 和 15 GR。
Capri (*) helps clear thicket 1 for 8 days then thicket 2 for 3 days.
Capri(*) 用了 8 天协助清洁 1 号灌木丛，然后又用了 3 天协助清洁 2 号灌木丛。

English version 英文版本：

	Thicket 1 (20)		Thicket 2 (15)	
Day	Cleared	Rem.	Cleared	Rem.
1–8	16*	4	8	7
9–11	3	1	6*	1
12	1	0	1	0

Chinese version 中文版本：

	1 号灌木丛(20)		2 号灌木丛(15)	
天数	已清洁	待清洁	已清洁	待清洁
1–8	16*	4	8	7
9–11	3	1	6*	1
12	1	0	1	0

12 days are required to clear the thickets.
所有灌木丛清洁完毕需要 12 天。

(Two tables in each example are same, but in different languages)
（同一案例中的两个表格内容相同，仅为方便不同语言同学使用）

For each of the following, find the minimum number of days required to clear all the thickets.
求下列各项中所有灌木丛清洁完毕所需的最短天数。

A. Three thickets, of areas 40, 22 and 16 GR.
三片灌木丛，面积分别为 40、22、16 GR

B. Three thickets, of areas 30, 25 and 22 GR.
三片灌木丛，面积分别为 30、25、22 GR

C. Four thickets, of areas 47, 37, 27 and 17 GR.
四片灌木丛，面积分别为 47、37、27、17 GR

Part C: Prize Questions 1–2

This section has two optional prize questions. They are not part of the core competition and do not contribute to your overall score. You can still get a perfect score without attempting them.

Results for these questions will only be used in determining prize winners. If you would like to be considered for a prize, you are invited to attempt these questions. Prize questions should only be attempted after you have completed your responses for Questions 1–9.

Each prize question has two parts and has the same rules as a core question in the paper.

Note: the full introductory text is copied below but the examples are omitted.

Prize 1. Multiswap

You have a line of ✓s and ✗s. You want all of the ✓s to be on the left and the ✗s to be on the right.

You will do this by several rounds of swapping. In each round:

1. a move consists of swapping a ✓with the ✗ immediately before it
2. there can be several moves in a round
3. neither a ✓ nor a ✗ can be part of more than one move in a round.

A. The following line has 36 ✗s and 36 ✓s.

$$\overbrace{\text{✗} \cdots \text{✗}}^{10}\ \overbrace{\text{✓} \cdots \text{✓}}^{10}\ \overbrace{\text{✗} \cdots \text{✗}}^{12}\ \overbrace{\text{✓} \cdots \text{✓}}^{12}\ \overbrace{\text{✗} \cdots \text{✗}}^{8}\ \overbrace{\text{✓} \cdots \text{✓}}^{8}\ \overbrace{\text{✗} \cdots \text{✗}}^{6}\ \overbrace{\text{✓} \cdots \text{✓}}^{6}$$

How many rounds would it take to move all of the ✓s to the left?

This would give $\overbrace{\text{✓} \cdots \text{✓}}^{36}\ \overbrace{\text{✗} \cdots \text{✗}}^{36}$

B. The following line has 26 ✗s and 21 ✓s.

$$\overbrace{\text{✗} \cdots \text{✗}}^{8}\ \overbrace{\text{✓} \cdots \text{✓}}^{9}\ \overbrace{\text{✗} \cdots \text{✗}}^{7}\ \overbrace{\text{✓} \cdots \text{✓}}^{4}\ \overbrace{\text{✗} \cdots \text{✗}}^{8}\ \overbrace{\text{✓} \cdots \text{✓}}^{5}\ \overbrace{\text{✗} \cdots \text{✗}}^{3}\ \overbrace{\text{✓} \cdots \text{✓}}^{3}$$

How many rounds would it take to move all of the ✓s to the left?

This would give $\overbrace{\text{✓} \cdots \text{✓}}^{21}\ \overbrace{\text{✗} \cdots \text{✗}}^{26}$

Prize 2. Capri

You are using your goats to clear blackberry thickets. Your goats are contrary creatures and can't be told what to do.

你正在驱赶山羊去清洁黑莓灌木丛，但这些山羊都很叛逆，并不会完全听从你的指令。

1. Goats will only work for whole days.

   山羊的工作时间只能按整天计算。

2. If a goat has been assigned to a thicket, it will not permit any other goat, except Capri, help it clear the thicket.

   如果某只山羊被分配到某个灌木丛，那么这只山羊不会允许除 Capri 之外的其他山羊来帮它清理这个灌木丛。

3. Each goat can clear an area of 1 GoatRood (GR) of blackberries per day.

   每只山羊每天只能清理 1 GR 的黑莓。

4. Capri:

   i. Capri will not work by himself.
   Capri 不会单独工作。

   ii. The other goats will let Capri join them.
   其他山羊会邀请 Capri 一起工作。

   iii. If Capri helps another goat, they clear 2 GR of blackberries per day.
   如果 Capri 帮助其他山羊一起清洁，那么这两只山羊一天就能清洁 2 GR 的黑莓。

Each of the other goats has been assigned a blackberry thicket. You know the area of each thicket, and therefore how many days one goat would take to clear it. You want to use Capri so that all thickets are cleared in as few days as possible.

除 Capri 之外的每一只山羊都被分配了一片黑莓灌木丛。你了解每片灌木丛的面积，所以可以计算出每只山羊要花多少天的时间才能清理完它们被分配到的灌木丛。现在你想派出 Capri 参加清洁任务，这样一来就可以将所有灌木丛的总清洁时间尽可能地缩短。（下表展示两个分别需要 6 天和 12 天才能清洁完毕的案例）

A. How long would it take to clear six thickets, of areas 56, 50, 48, 42, 39 and 37 GR?

B. How long would it take to clear seven thickets, of areas 87, 78, 72, 67, 63, 59 and 54 GR?

Solutions

Part A: Questions 1–6

1. Flow

There are two key shortcuts:

- the [even?] —YES→ [$\div 2$] loop:
  this finds the largest odd factor of a number.

- the [≥ 100?] —YES→ [-100] loop:
  this finds the last two digits of a number (mod 100).

Tracing through the flow chart we get the sequence of numbers in the following table, reading top to bottom then left to right:

Input	38			
Reverse	83	94	14	12
Odd factor	83	47	7	3
Triple	249	141	21	9
Mod 100	49	41	21	9
Output				9

So the ‘reverse the digits’ process is applied 4 times in all.

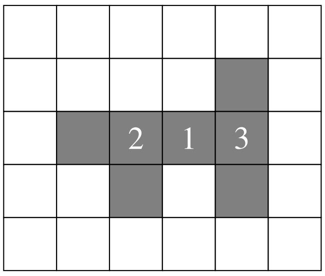
Hence (D).

2. Growth

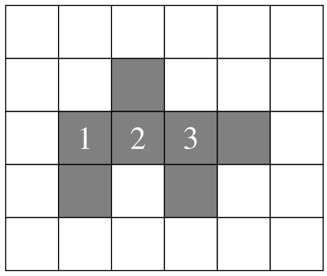
We make the following observations:

1. If there is exactly one square with 2 neighbours, it must have been the first square chosen.
2. If there is no 2×2 block:
   (a) If there is more than one square with 2 neighbours, the final diagram is not possible.
3. If there is a 2×2 block:
   (a) If there is exactly one square with 2 neighbours outside the block, it must have been the first square chosen.
   (b) If there is more than one square with 2 neighbours outside the block, the final diagram is not possible.
   (c) If there are two non-adjacent squares with exactly 2 neighbours in the block, either could be the first square chosen.

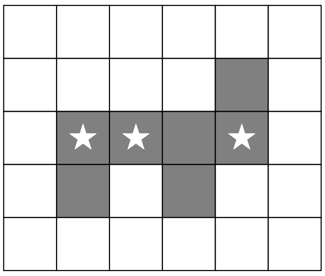
We use these rules to determine the order in which squares were selected.



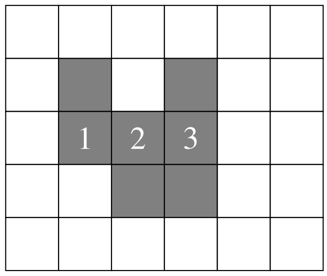
✓ rule 1



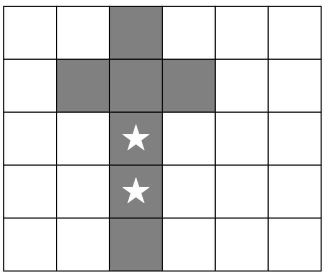
✓ rule 1



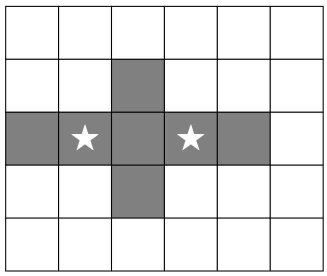
✗ rule 2(a)



✓ rule 3(a)



✗ rule 2(a)



✗ rule 2(a)

3 diagrams are possible.

Hence (C).

3. Magnetic drone

$2 \rightarrow \ 2\uparrow$ does not get from one island to another,

$2 \leftarrow \ 3\uparrow$ is the instruction for D to A,

$2 \rightarrow \ 1\downarrow$ is the instruction for E to B,

$2 \leftarrow \ 1\uparrow$ is the instruction for B to E,

$1 \rightarrow \ 2\downarrow$ is the instruction for B to D,

$1 \leftarrow \ 2\downarrow$ is the instruction for A to C.

As there is no instruction from C to any other island, C is the end point. Thus, working backwards, the sequence is EBDAC.

Hence (E).

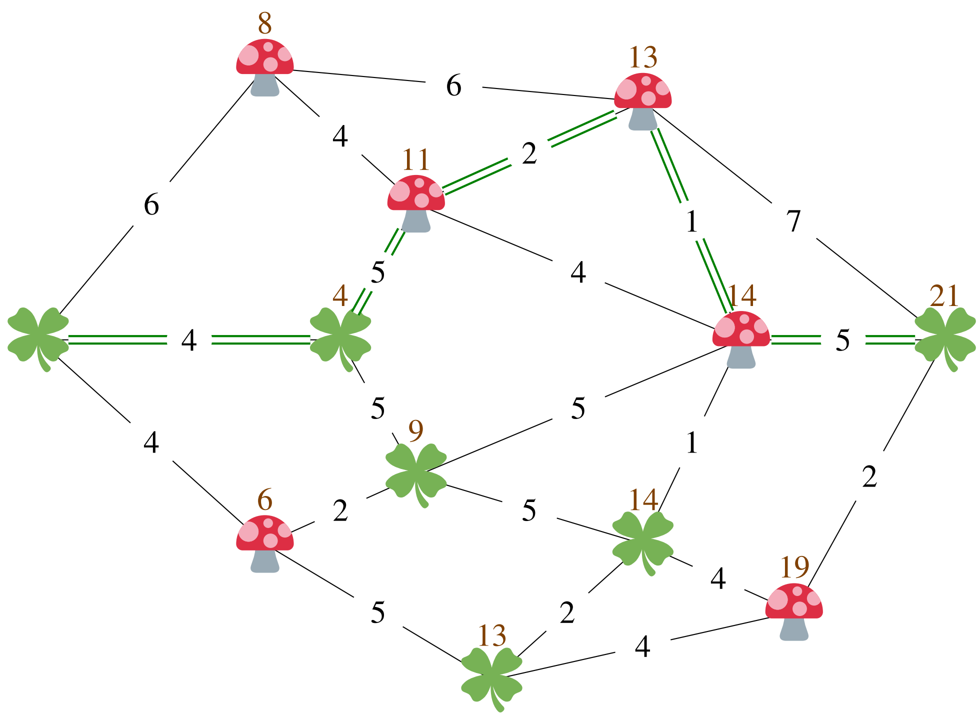
4. Avalon

It is clear that going right to left will only increase the time. You will always go left to right.

Solution 1

In the map below, the number above each wood or glen gives the earliest time in hours that you could leave it in the outfit you needed to be in while travelling through it. For instance it will take you 6 hours to reach the top left glen and another 2 hours to change your outfit.

Where a wood or glen could be reached from more than one direction, you will come from the direction allows you to enter it in the correct outfit earliest.



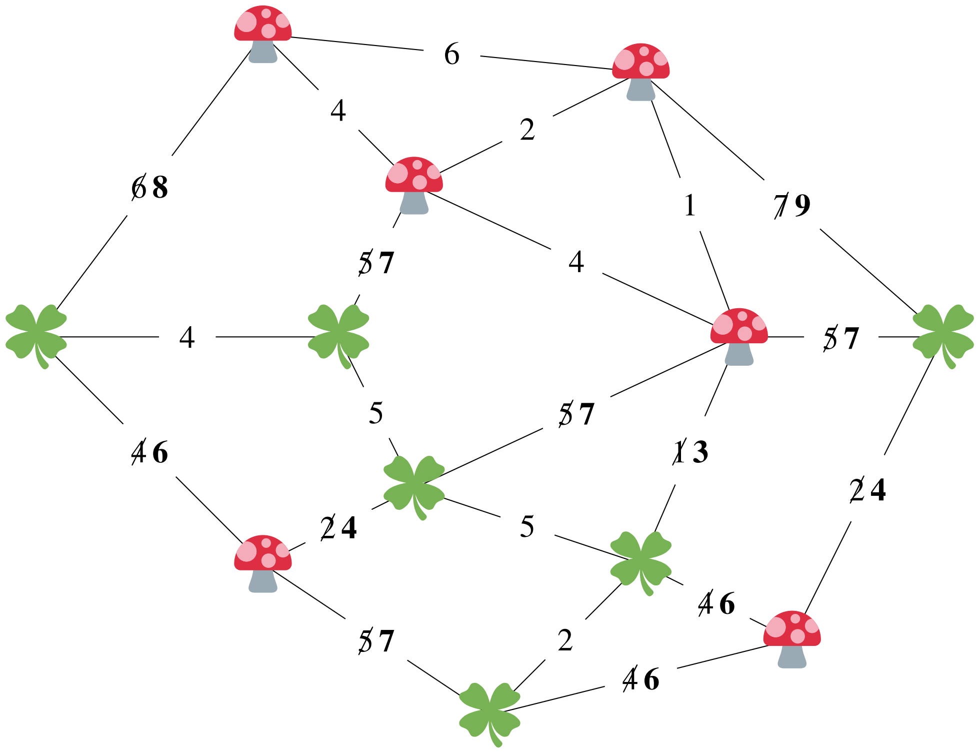
The double green line shows the route that results in you finishing your journey in the least time. This takes 21 hours.

Hence (C).

Solution 2

In the first solution we added 2 hours to the entry time to a wood or glen if we left the previous wood or glen in the wrong outfit. Consider the glen directly to the left of the final wood. It can be approached from four directions, two via a wood and two via a glen. This means that every time we determine the entry time we have to decide whether to add the 2 hours.

A simpler approach is to add the 2 hours to the time taken between a wood and a glen, and between a glen and a wood. This gives us the following map.



We can now use the method in *Solution 1* without having to worry about whether or not to add the 2 hours.

5. Row delete

Solution 1

We first find out the effect of the sequence $3, 1, 4, 1, 5, 9, 2, 6, 5$.

Start by listing the row numbers 1, 2, 3, ... in order. The first 3 in the sequence means cross out the 3rd number, which is 3. The 1 means cross out the first number, which is 1. The 4 means cross out the 4th remaining number, which at this point is 6. So after three steps we have the following:

~~1~~ 2 ~~3~~ 4 5 ~~6~~ 7 ...

For each new row number n, cross out the nth remaining number in the list. If at any point additional numbers are needed, just add them to the end. After the full sequence we are left with the following:

~~1~~ ~~2~~ ~~3~~ 4 ~~5~~ ~~6~~ 7 8 ~~9~~ 10 ~~11~~ ~~12~~ 13 ~~14~~ 15 ...

So we need to find the cheapest sequence which ultimately deletes the same collection of rows.

Starting at the left end, we can delete rows 1, 2 and 3 with $1, 1, 1$ and clearly this is the cheapest strategy that does this:

~~1~~ ~~2~~ ~~3~~ 4 5 6 7 ...

In order to leave 4 and 7 alone and cross out 5 and 6, we proceed with $2,2$:

$$\not{1} \quad \not{2} \quad \not{3} \quad 4 \quad \not{5} \quad \not{6} \quad 7 \quad \ldots$$

Continuing in this way, we find that the cheapest sequence is $1,1,1,2,2,4,5,5,6$ which has cost

$$1+1+1+2+2+4+5+5+6 = 27.$$

Hence (D).

Solution 2

We have to delete the rows originally numbered 1, 2, 3, 5, 6, 9, 11, 12 and 14.
Clearly we will delete them in that order.

The 1 costs 1.
It will cost $2-1=1$ to delete the 2.
It will cost $3-2=1$ to delete the 3.
It will cost $5-3=2$ to delete the 5.

Continuing in this way we have
$1+(2-1)+(3-2)+(5-3)+(6-4)+(9-5)+(11-6)+(12-7)+(14-8)$
$=(1+2+3+5+6+9+11+12+14)-(1+2+3+4+5+6+7+8)$
$=(1+2+3+5+6+9+11+12+14)-((8\times 9)\div 2)$
$=63-36=27$

Hence (D).

6. Card choices

We will use the notation $n(i)$ for the card numbers, and $t(i)$ for the largest total of the first i numbers subject to the given rules.

We will write the card numbers in a line, with a | to indicate the pairs.

5 1 | 4 5 | 3 9 | 5 7 | 7 10 | 8 9

Then $t(i) = n(i) + t(j)$ where $t(j)$ is the largest t value for all $j < i$. So for our data, $t(1) = n(1) = 5$, $t(2) = n(2) = 1$. Then $t(3) = n(3) + t(2) = 4+1 = 5$, etc.

$n(i)$	5	1	4	5	3	9	5	7	7	10	8	9
$t(i)$	5	1	5	6	4	15	10	13	17	25	25	26

The largest total following the rules is 26 $(1+4+5+7+9)$, by choosing the numbers circled below.

5	④	3	⑤	⑦	8
①	5	9	7	10	⑨

Hence (C).

Part B: Questions 7–9

7. Multiswap

Consider ✗ ✗ ✓ ✓ ✓

round	moves	:	giving
1	We can only swap the first ✓	:	✗ ✓ ✗ ✓ ✓
2	We swap the first two ✓s	:	✓ ✗ ✓ ✗ ✓
3	We swap the 2^{nd} and 3^{rd} ✓s	:	✓ ✓ ✗ ✓ ✗
4	We swap the 3^{rd} ✓	:	✓ ✓ ✓ ✗ ✗

This establishes the pattern. If there are c ✗s ahead of several ✓s, it will take c rounds to move the first ✓ahead of the ✗s, and extra round to move the second ✓, and so on.

A. ✗ ✗ ✗ ✓ ✓ ✓ ✓ ✓

The leftmost ✓ will take 3 rounds to move past the 3 ✗s.

The second from the left will not take part in the first round, then will take another 3 rounds to swap with the 3 ✗s, for a total of 4 rounds.

Similarly the third ✓will take 5 rounds, the fourth 6 rounds and the fifth 7 rounds.

Hence 7.

B. ✗ ✗ ✓ ✗ ✗ ✗ ✗ ✓ ✓ ✓ ✓

The leftmost ✓ will take 2 rounds to move past the 2 ✗s.

The first ✓in the second group will take $2+4=6$ rounds to move past all of the ✗s, by which time the first ✓ has moved to the left of the line.

The remaining 3 ✓s will take anther 3 rounds, for a total of $6+3=9$ rounds.

Hence 9.

C. ✗ ✗ ✓ ✓ ✓ ✓ ✗ ✗ ✓ ✓ ✓ ✗ ✗ ✓

The first ✓ in the second group will take $2+2=4$ rounds to move past all of the ✗s.

However the rightmost ✓ in the first group will take $2+3=5$ rounds to move past the ✗s ahead of it.

So the first ✓ in the second group can't be in position until after 6 rounds and the rightmost ✓ will need another 2 rounds, making 8 rounds.

The final tick has 6 ✗s ahead of it so would need 6 rounds to move past them. However the previous ✓ needs 8 rounds so the final ✓ needs a 9^{th} round.

Hence 9.

8. Capri

We will use $t_1 \leq t_2 \ldots$ for the number of days to clear thicket 1, thicket 2, ...

Solution 1

Two thickets

Capri only helps clear thicket 1.
This is the case where $t_2 \leq t_1/2$

Capri helps clear both thickets.
This is the case where $t_2 > t_1/2$

- Capri will help with thicket 1 until both thickets have the same time remaining.
- Then he will help clear both thickets equally.

Note that once both thickets have 2 days left, it does not reduce the time to clear both if Capri helps with one of them.

Three thickets

Capri only helps clear thicket 1.
This is the case where $t_2 \leq t_1/2$

Capri helps clear thickets 1 and 2.
This is the case where $t_2 > t_1/2$ and $t_3 \leq t_1/2$.

- Capri will help with thicket 1 until both thickets have the same time remaining.
- Then he will help clear both thickets equally.

Capri helps clear all three thickets.
This is the case where $t_3 > t_1/2$

- Capri will help with thicket 1 until thickets 1 and 2 have the same time remaining.
- Then he will help clear thickets 1 and 2 equally until they have the same time remaining as thicket 3.
- Then he will help clear all thickets equally.

Note that once all the thickets have 2 or 3 days left, it does not reduce the time to clear them all if Capri helps.

We can now solve parts A and B.

A.

	Thicket 1 (40)		Thicket 2 (22)		Thicket 3 (16)	
Day	Cleared	Rem.	Cleared	Rem.	Cleared	Rem.
1-16	32*	8	16	6	16	0
17-18	4*	4	2	4		
19	2*	2	1	3		
20	1	1	2*	1		
21	1	0	1	0		

The thickets will be cleared in 21 days.

Capri helps with the first two thickets.

B.

	Thicket 1 (30)		Thicket 2 (25)		Thicket 3 (22)	
Day	Cleared	Rem.	Cleared	Rem.	Cleared	Rem.
1-5	10*	20	5	20	5	17
6-8	6*	14	3	17	3	14
9-11	3	11	6*	11	3	11
12-13	4*	7	2	9	2	9
14-15	2	5	4*	5	2	7
16-17	2	3	2	3	4*	3
18-20	3	0	3	0	3	0

The thickets will be cleared in 20 days.

Capri helps with all three thickets.

Four thickets

The pattern is now established:

Check to see whether Capri spends all of his time helping clear thicket 1.
- If he does, we are finished
- If not, even up thickets 1 and 2.
  - If thicket 3 is finished, Capri helps thickets 1 and 2 equally.
  - If not, even up thickets 1, 2 and 3.
    - If thicket 3 is finished, Capri helps thickets 1, 2 and 3 equally.
    - If not, even up thickets 1, 2 and 3 and then Capri helps all thickets equally.
      - ...

We can now apply this procedure to the remaining data in the question.

C.

	Thicket 1 (47)		Thicket 2 (37)		Thicket 3 (27)		Thicket 4 (17)	
Day	Cleared	Rem.	Cleared	Rem.	Cleared	Rem.	Cleared	Rem.
1-10	20*	27	10	27	10	17	10	7
11-17	14*	13	7	20	7	10	7	0
18-19	4*	9	2	18	2	8		
20-27	8	1	16*	2	8	0		
28	1	0	2*	0				

The thickets will be cleared in 28 days.

Capri helps with the first two thickets.

Solution 2

We can short-cut the above solutions by doing a little analysis.

If Capri only helped with thicket 1, it would take $t_1/2$ (rounded up) days to clear the thicket.

(We need to round up to allow for t_1 being odd, in which case Capri would not help on the last day.)

If by this time thicket 2 (and therefore all other thickets) has been cleared, Capri will only help with thicket 1. This gives us our first step.

- If $ceiling(t_1/2) \geq t_2$ then the time to clear all thickets is $ceiling(t_1/2)$.
  ($ceiling$ is the rounding-up function.)

If $ceiling(t_1/2) < t_2$, then Capri should help with thicket 2 as well. This will take $(t_1+t_2)/3$ days. If this is $\geq t_3$ then Capri will just help with thickets 1 and 2.

It will be useful to introduce a $helpsWith_i$ function. This is the time it would take to clear the first i thickets with Capri's help.

$helpsWith_1 = ceiling(t_1/2)$
$helpsWith_2 = ceiling((t_1+t_2)/3)$
$helpsWith_3 = ceiling((t_1+t_2+t_3)/4)$
...

We can now extend the approach above

	condition	days to clear
	$helpsWith_1 \geq t_2$	$helpsWith_1$
else	$helpsWith_2 \geq t_3$	$helpsWith_2$
else	$helpsWith_3 \geq t_4$	$helpsWith_3$
else	...	

We can now use this approach with the thickets in the question.

A. 3 thickets, taking 40, 22 and 16 days.
$helpsWith_1 = 20,$ which is $< t_2,$

$helpsWith_2 = 21,$ which is $> t_3$,
Hence $helpsWith_2 = 21$ days.
Capri helps with 2 thickets.

B. 3 thickets, taking 30, 25 and 22 days.
$helpsWith_1 = 15,$ which is $< t_2$,
$helpsWith_2 = 19,$ which is $< t_3$,
Hence $helpsWith_3 = 20$ days.
Capri helps with 3 thickets.

C. 4 thickets, taking 47, 37, 27 and 17 days.
$helpsWith_1 = 24,$ which is $< t_2$,
$helpsWith_2 = 28,$ which is $> t_3$,
Hence $helpsWith_2 = 28$ days.
Capri helps with 2 thickets.

9. Bus passengers

We will refer to the passengers who got on at A as ‘the originals’, and those who got on later as ‘the subsequents’.

Our aim is to have as many originals as possible on the bus at each leg. So we assume that when passengers get off, as many as possible are subsequents, keeping in mind the requirement that every passenger must complete at least one leg. We only need to keep track of the *number* of originals and the *number* of subsequents on the bus for each leg.

A.

Stop	A		1		2		3		4		5		6		B
On	20		10		0		5		0		5		0		–
Off	–		0		8		0		10		0		3		19
Originals		20		20		20		20		17 20-3		17		17	
Subsequents		0		10 0+10		2 10-8		7 2+5		0 7-7		5 0+5		2 5-3	

At most 17 passengers could have travelled from A to B.

B.

Stop	A		1		2		3		4		5		B
On	20		10		8		4		4		2		–
Off	–		0		8		6		8		4		22
Originals		20		20		20		20		20		20	
Subsequents		0		10		10 (10-8)+8		8 (4-6)+10		4 (4-8)+8		2 (2-4)+4	

All 20 passengers could have travelled from A to B.

C.

Stop	A		1		2		3		4		5		6		B
On	20		5		6		4		8		5		5		–
Off	–		8		3		6		10		7		7		12
Originals		20		12 20-8		12		12		8 12-(10-6)		8		7 8-(7-6)	
Subsequents		0		5		8 (5-3)+6		6 (8-6)+4		8 (6-6)+8		6 (8-7)+5		5 (6-6)+5	

At most 7 passengers could have travelled from A to B.