Part A: Questions 1-6

Each question should be answered by a single choice from A to E.
每题有五个选项，考生应从中选出一个正确答案。
Questions are worth 3 marks each.
每题 3 分。

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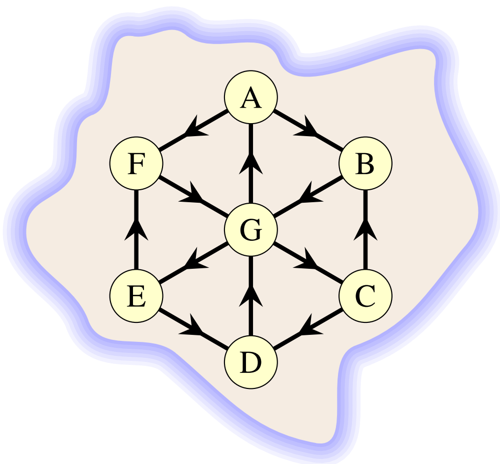
1. Octave Island-奥克塔夫岛

Octave Island has seven small towns labelled A to G. The towns are connected by one-way roads as shown.

岛上有七个小镇，分别用字母 A 到 G 进行标记。各小镇之间通过单行道相连，如图所示。



Mabel lives in A and needs to visit G, E and D in order, then return to A. But she can't drive directly from A to G because of the one-way roads. Each road takes 1 hour to travel on, so her return journey takes 6 hours.

住在 A 小镇的 Mabel 想按照 G、E、D 的顺序去这三个小镇旅游，然后再回到 A 小镇。但由于各小镇之间都是单行道，她无法直接从 A 小镇去到 G 小镇。每条单行道的行驶时间为 1 小时，Mabel 往返路程一共耗时 6 小时。

Mabel's friend Klara lives in D and wants to visit the towns F, C and A in order and return to D.

Mabel 的朋友 Klara 住在 D 小镇，想按照 F、C、A 的顺序去这三个小镇旅游，然后再回到 D 小镇。

What is the shortest time, in hours, that Klara could take for her return journey?

请问 Klara 往返路程最少需要几个小时？

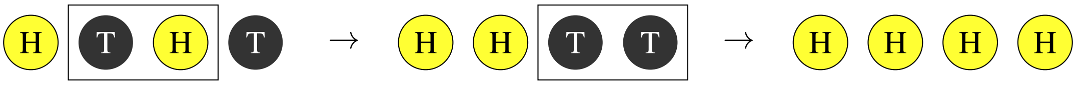
(A) 10 (B) 11 (C) 12 (D) 13 (E) 14

2. Heads up-翻面

You have a line of coins, some with the head side up and some with the tail side up. You want all of the coins to have the head side up. The only move you are allowed is to flip two adjacent coins. You will apply this move repeatedly until all the coins have their head sides up.

一排硬币中，有些正面（H）朝上，有些背面（T）朝上。此时，你想让一排硬币全部正面（H）朝上。操作过程中，每轮只能同时翻转两个相邻硬币，一直翻转直到所有硬币都正面（H）朝上为止。

Example 举例



You are given the following line:
给定如下一排硬币：

What is the smallest number of moves to have all of the coins with their head sides up?

请问至少需要翻转几轮才能使所有硬币正面（H）朝上？

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

3. Growth-野蛮生长

The game of *Growth* takes place on a grid of white squares.

野蛮生长 游戏采用一个由白色方格组成的网格。

- To start, a single square somewhere in the grid is chosen. It is shaded.

  游戏开始时，网格中随机出现一个阴影方格。

- To make a move, a player chooses a shaded square and shades two white squares that are adjacent to it. (Adjacent means the squares have an edge in common.)

  玩家每一轮需要选择一个阴影方格，然后为其两个相邻白色方格涂上阴影。(相邻方格指的是共用一条边的方格。)

- A shaded square **cannot** be chosen if it does not have two white squares adjacent to it.

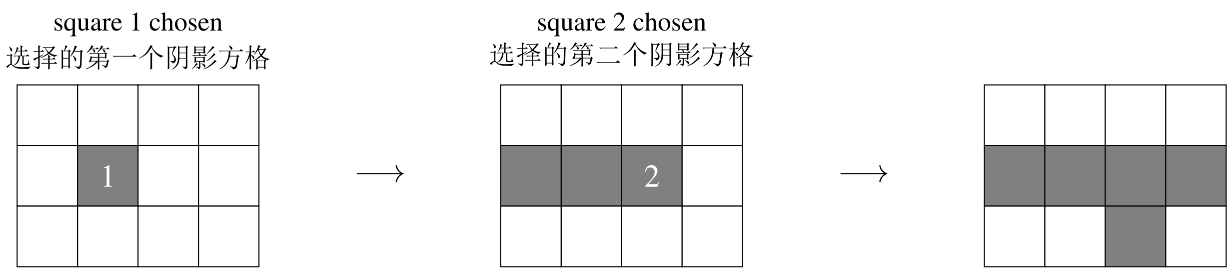
  如果一个阴影方格没有两个相邻的白色方格，那么它就**不能**被选择。

- A shaded square **can** be chosen more than once.

  一个阴影方格**可以**被多次选择。

The example below shows the first shaded square and how the board *could* develop over two moves on a 4×3 grid.

下图显示了在这个 4×3 网格中选择的第一个阴影方格以及经过两轮操作后网格可能的变化情况。

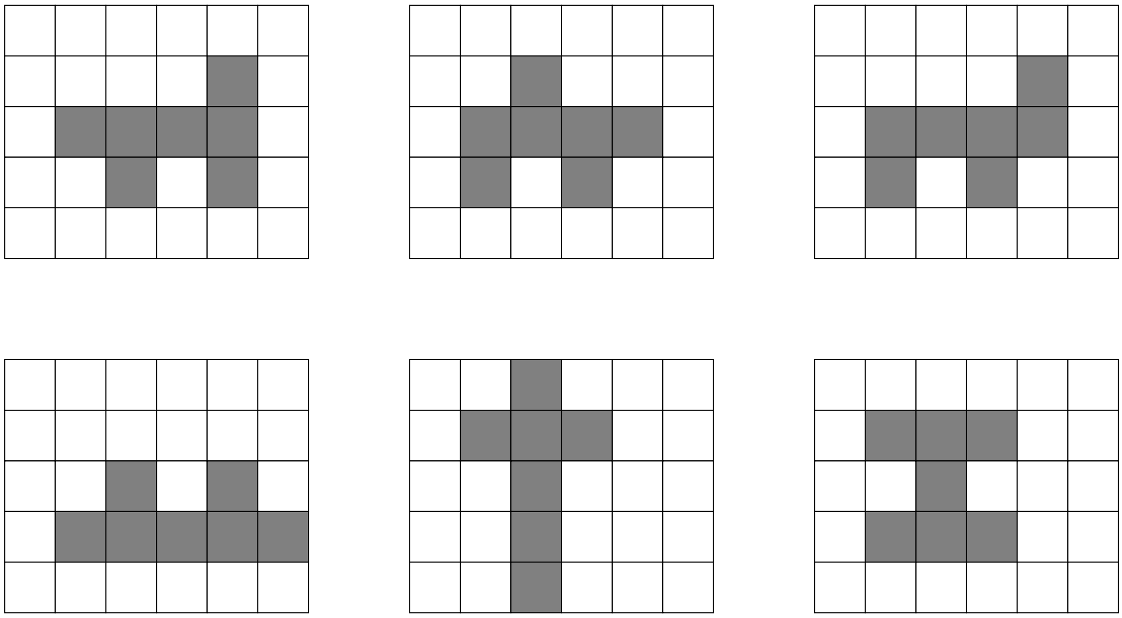


A game is played on a 6×5 grid.

如果该游戏采用的是一个 6×5 的网格。

How many of the following diagrams are possible after three moves?

请问在下列图片中，有多少张图片显示的是三轮操作之后的结果？



(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

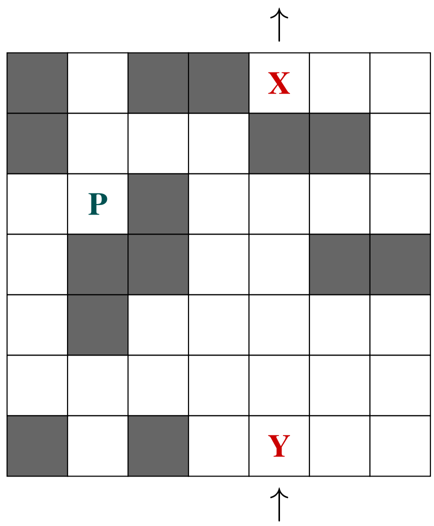
4. Donut Prince-甜甜圈王子

The game *Donut Prince* is played on the 7×7 grid shown. The prince can move one square up, down, left or right on each turn. He cannot move onto a solid grey square and diagonal moves are not allowed.

甜甜圈王子 游戏采用如下的 7×7 网格。王子每轮都可以在上、下、左、右四个方向上移动一个方格。但是，他不能进入灰色方格，也不能在对角线方向上移动。

When the prince is at any of the four edges of the grid, one more move towards that edge transports him to the same position on the opposite side. For example, he can get from square **X** to square **Y** with one move up.

当王子位于该网格四条边中任意一边时，朝着所在边方向再前进一步可以被传送到对边的同一位置。例如，王子可以从 **X** 方格向上移动一步到达 **Y** 方格。



The prince starts at square **P**.
现在王子从 **P** 方格出发。

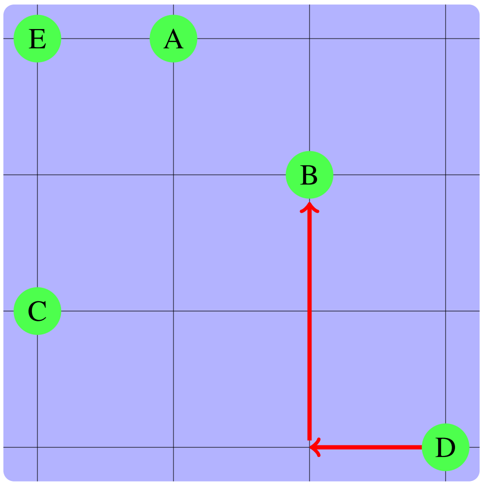
How many of the white squares could *not* be reached by the prince in 6 moves or fewer?

请问经过 6 次或者更少次数的移动后，王子 并未 涉足的白色方格有多少个？

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

5. Magnetic drone-磁力飞侠

Magnetic drones can only fly along magnetic field lines or perpendicular to them. Their instructions are of the form $m \rightarrow \ n \downarrow$.
磁吸无人机只能沿着磁场边线或垂直于这些边线飞行。该无人机指令形式为 $m \rightarrow \ n \downarrow$。



For instance, the instruction $1 \leftarrow \ 2 \uparrow$ could be used to fly from island D to island B.
例如，无人机执行指令 $1 \leftarrow \ 2 \uparrow$，可从 D 岛飞往 B 岛。

David's drone started on one of the islands, but we don't know which one. It then flew to three other islands and landed on the fourth. Thus it spent time on all of the islands exactly once, and did not return to its starting point.
David 的无人机从其中某个岛屿起飞，经过其他三个岛屿后，在第四个岛屿降落。此次飞行中，这架无人机对所有岛屿都恰好进行了一次飞越，并且它并未返回起飞岛屿。

To carry out this tour, it used exactly four of the instructions below in some order.
此次飞行，无人机恰好按某种顺序执行了以下指令中的四种指令。

$2 \rightarrow \ 2 \uparrow$

$2 \leftarrow \ 3 \uparrow$

$2 \rightarrow \ 1 \downarrow$

$2 \leftarrow \ 1 \uparrow$

$1 \rightarrow \ 2 \downarrow$

$1 \leftarrow \ 2 \downarrow$

On which island did the drone start?
请问无人机是从哪个岛屿起飞的？

(A) A (B) B (C) C (D) D (E) E

6. Square-sum-平方-和

A *square-sum sequence* is a list of numbers where pairs of adjacent numbers add to a perfect square.

平方-和数列由一系列数字构成，其中两个相邻数字相加等于一个完全平方数。

- 7, 2, 14 is a square-sum sequence because $7+2=9=3^2$ and $2+14=16=4^2$.

  7, 2, 14 是一个平方-和数列，因为 $7+2=9=3^2$；$2+14=16=4^2$ 都等于一个完全平方数。

- 7, 14, 2 is *not* a square-sum sequence because $7+14=21$, which is not a square.

  7, 14, 2 不是一个平方-和数列，因为 $7+14=21$ 不是一个完全平方数。

- Note that the reverse of any square-sum sequence is also a square-sum sequence.
  注意，任何一个平方-和数列从后往前排列也是一个平方-和数列。

  For example, 14, 2, 7 is a square-sum sequence.

  例如，14、2、7 就是一个平方-和数列。

You have been given the nine numbers 2, 3, 6, 8, 17, 19, 30, 34, 47.

给定九个数字 2、3、6、8、17、19、30、34、47。

Your task is to arrange them into a square-sum sequence.

现在需要对这九个数字进行排列组合，形成一个平方-和数列。

After you have arranged them, how many numbers are between the 19 and the 47?

请问通过排列组合得到的这个平方-和数列中，数字 19 和 47 之间有多少个数字？

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Part B: Questions 7–9

Each question has three parts, each of which is worth 2 marks.
每题有三个部分，每部分 2 分。
Each part should be answered by a number in the range 0–999.
每部分答案应为一个介于 **0–999** 之间的数字。

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7. Plantings-苗床种植

Elle wants to plant her rural block with banksias (B), grevilleas (G) and waratahs (W). She has marked out several rectangular garden beds. She wants to plant out her garden beds so that:

Elle 想要在农村种植山茂樱（B）、银桦木（G）、瓦松（W）。她已经将土地规划成多块矩形苗床，并且计划按照以下模式在苗床中栽种：

- each garden bed will be planted with only one type of plant – banksias or grevilleas or waratahs

  每块苗床只栽种一种植物——山茂樱（B）、银桦木（G）或瓦松（W）
- neighbouring garden beds will not be planted with the same type of plant (neighbouring garden beds are those that have an edge or part of an edge in common, but not just a corner).

  相邻苗床中栽种的植物类型不同（相邻苗床指的是共用一条边或者共用一部分边的苗床。注意，仅共用一个角落的苗床不是相邻苗床。）

Just as she is pondering the possibilities, her mother announces that she has just put some plants in! Luckily the rules Elle set have been obeyed, and now there is no decision to make: there is only one possible way to plant the rest of the beds.

正当 Elle 考虑计划可行性时，她的妈妈说她已经在一些苗床里面种了一些植物了！不过，幸好妈妈遵循了 Elle 设定的栽种规则，现在 Elle 只能在剩下未种植的苗床中栽种植物了。

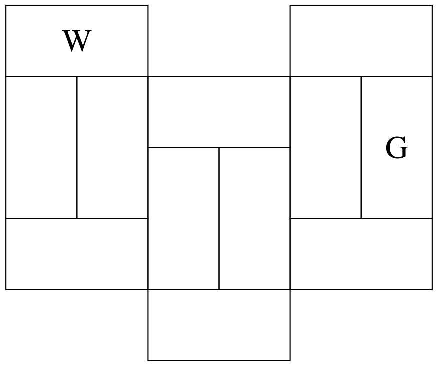
For each of the rural blocks below, find the number of beds that are planted with each type of plant. *This includes the beds Elle's mother has planted.*

对于下列各块农村土地，求每种植物的苗床数量。其中包含 *Elle* 妈妈栽种的苗床。

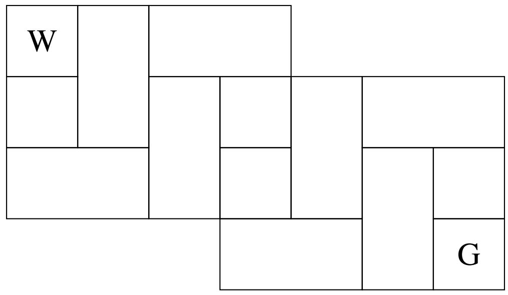
Your answer will be a 3-digit number, giving the number of beds planted with banksias, grevilleas and waratahs. For instance 123 would mean 1 bed planted with banksias, 2 with grevilleas and 3 with waratahs.

你的答案应该为一个 3位数，分别表示山茂樫苗床、银桦木苗床、瓦松苗床的数量。例如数字123表示 1 个山茂樫苗床、2 个银桦木苗床和 3 个瓦松苗床。

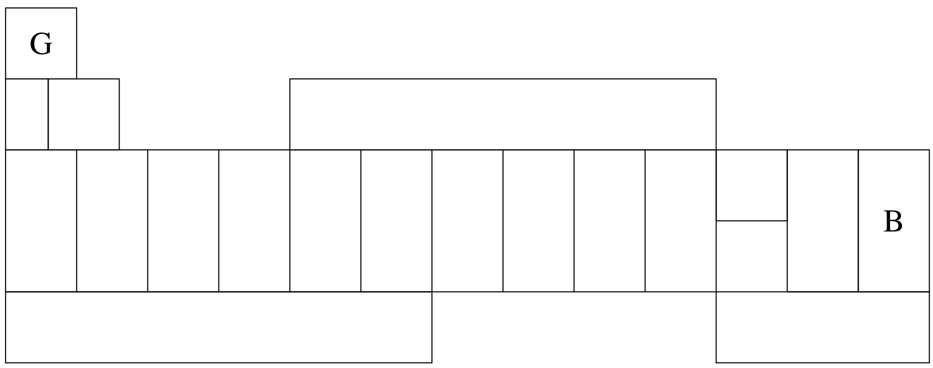
A.



B.



C.

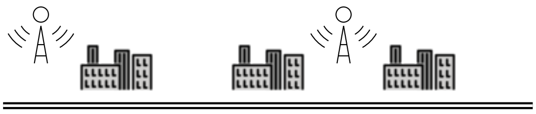


8. Communication towers-通信塔

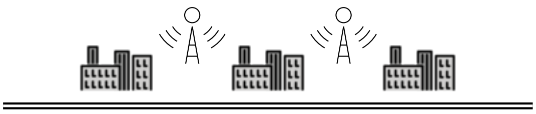
There are several towns along a road. Sites have been identified for communication towers to service the towns. To have service, each town must be next to one or two communication towers.

沿路有几座通信塔。搭建通信塔是为了给城市带来更好的通信保障，因此必须确保每个小镇旁边有一座或两座通信塔。

Examples 举例:



✓



✓

✗ third town misses out
第三座小镇旁边没有通信塔

✗ second town misses out
第二座小镇旁边没有通信塔

You know the cost of building a communication tower on each site. You want the total cost of building the communication towers to be as low as possible.

基于你所掌握的在各个位置搭建通信塔的成本，现在你想将所有通信塔的总体搭建成本尽可能降到最低。

For each of the following, what is the lowest cost to build communication towers so that every town is next to at least one tower?
对于下列每一种通信塔搭建方案，计算可以使得每个小镇旁边都至少有一座通信塔的最低成本。

(Each number represents the cost of building a tower on that site. The Ts represent towns.)（其中各个数字代表在该位置搭建一座通信塔的成本。T 代表小镇。）

A. 1 2 5 3 2 8 6 5 6 1
T T T T T T T T T

B. 3 2 1 2 5 3 2 3 4 5
T T T T T T T T T

C. 4 3 2 1 4 7 2 6 5 3 4 1
T T T T T T T T T T T

9. Bus passengers-公交乘客

A bus is travelling from A to B, with several stops along the way.

一辆公交车从 A 点开往 B 点，沿途有一些经停站点。

At each stop several passengers get on and get off, as shown in the following tables.

每到达一个经停点时，都有一些乘客上下车，如下列表所示。

Every passenger travels at least one leg. No-one gets on and off at the same stop.

每位乘客至少搭乘一站，且没有人在同一站点上下车。

For each of the following trips, what is the greatest number of passengers who could have travelled from A to B for the entire journey?

对于下列每种公交线路，计算可以从 A 点一直搭乘到 B 点下车的最大乘客数量。

A.

Stop 经停站点	A	1	2	3	4	5	6	B
Passengers on 上车人数	20	10	0	5	0	5	0	–
Passengers off 下车人数	–	0	8	0	10	0	3	19

B.

Stop 经停站点	A	1	2	3	4	B
Passengers on 上车人数	20	10	4	7	0	–
Passengers off 下车人数	–	0	8	10	5	18

C.

Stop 经停站点	A	1	2	3	4	5	6	B
Passengers on 上车人数	20	5	6	4	8	5	5	–
Passengers off 下车人数	–	8	3	6	10	7	7	12

Solutions

Part A: Questions 1–6

1. Octave Island

Travelling between the towns on the list will require going via other towns.

(E|A) means that Kara could go via E or A.

Towns		Roads
list	route	
D→F	D→G→(E|A)→F	3
F→C	F→G→C	2
C→A	C→(B|D)→G→A	3
A→D	A→(F|B)→G→(E|C)→D	4

Kara's journey will take a total of $3+2+3+4=12$ hours.

Hence (C).

2. Heads up

Solution 1

We make the following observations:

1 The options for flipping two coins are

$$\begin{array}{ccc} \mathrm{T\,T} & \rightarrow & \mathrm{H\,H} \\ \mathrm{T\,H} & \rightarrow & \mathrm{H\,T} \\ \mathrm{H\,T} & \rightarrow & \mathrm{T\,H} \\ \mathrm{H\,H} & \rightarrow & \mathrm{T\,T} \end{array}$$

In each case if you start with an even number of Ts you end with an even number of Ts. So if there was an even number of Ts in the line, there will always be an even number of Ts in the line and it will be possible to end up with no Ts.
(This is the case for the line in the question.)

2 If a coin is flipped with both of its neighbours, it does not matter in which order the flips are executed.

Observation 2 enables us to develop a left-to-right algorithm.
If the first coin is T, we flip the first two coins.
Then if the second coin is T, we flip the first and second coins, and so on.

So in this case the first coin is a T so the first two coins must be flipped.
This flips the second coin from an H to a T, so the second and third coins must be flipped.
This also flips the third coin from a T to an H, so the third and fourth coins do not need to be flipped.

In the diagram below, pairs of coins that need to be flipped are indicated by a $\frown$.

$$\mathrm{T} \frown \mathrm{H} \frown \mathrm{T} \quad \mathrm{H} \quad \mathrm{H} \quad \mathrm{T} \frown \mathrm{T} \quad \mathrm{T} \frown \mathrm{H} \frown \mathrm{H} \frown \mathrm{H} \frown \mathrm{H} \frown \mathrm{T}$$

Eight flips are required.
Hence (D).

Solution 2

Here we make a further observation

3 Consider the sequence of coins T H . . . H T with n Hs.
This can be be changed into H H . . . H H with $(n+1)$ flips.
$\underline{\mathrm{T\,H}}\,\mathrm{H\,T} \rightarrow \mathrm{H}\,\underline{\mathrm{T\,H}}\,\mathrm{T} \rightarrow \mathrm{H\,H}\,\underline{\mathrm{T\,T}} \rightarrow \mathrm{H\,H\,H\,H}$

We can use this observation to deduce the number of flips required without tracing through the algorithm.

$\mathrm{THTHHTTTHHHHT} = \mathrm{THT} \;\; \mathrm{HH} \;\; \mathrm{TT} \;\; \mathrm{THHHHT}$

T H T	requires 2 flips
T T	requires 1 flip
T H H H H T	requires 5 flips

Eight flips are required.

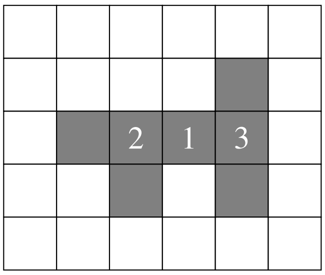
Hence (D).

3. Growth

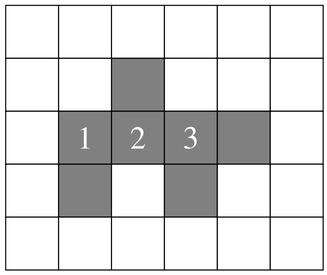
We make the following observations: 观察如下：

1. If there is exactly one square with 2 neighbours, it must have been the first square chosen.
2. If there is more than one square with 2 neighbours, the final diagram is not possible.

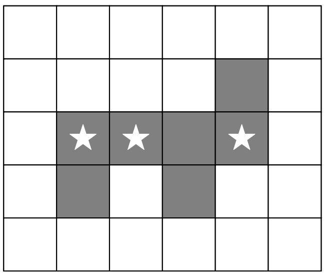
We use these rules to determine the order in which squares were selected.



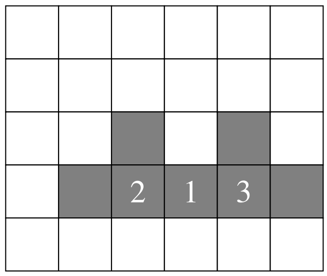
✓ rule 1



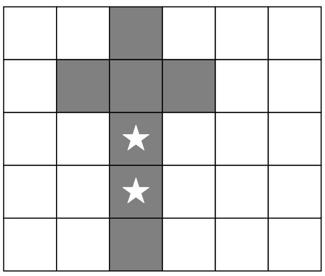
✓ rule 1



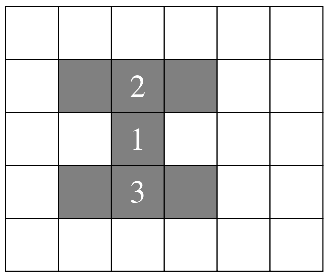
✗ rule 2



✓ rule 1



✗ rule 2



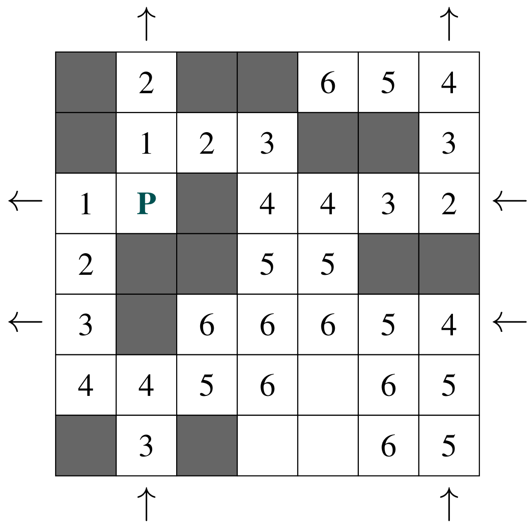
✓ rule 1

4 diagrams are possible.

Hence (D).

4. Donut Prince

We label the squares that the prince can get to with 1 move, then the squares he could get to in 2 moves, and so on.



There are 3 white squares that the prince could not reach in 6 moves.
Hence (B).

5. Magnetic drone

$2 \rightarrow \; 2 \uparrow$ does not get from one island to another,
$2 \leftarrow \; 3 \uparrow$ is the instruction for D to A,
$2 \rightarrow \; 1 \downarrow$ is the instruction for E to B,
$2 \leftarrow \; 1 \uparrow$ is the instruction for B to E,
$1 \rightarrow \; 2 \downarrow$ is the instruction for B to D,
$1 \leftarrow \; 2 \downarrow$ is the instruction for A to C.

As there is no instruction from C to any other island, C is the end point. Thus, working backwards, the sequence is EBDAC.
Hence (E).

6. Square-sum

We first build a table of the potential neighbours of each number.

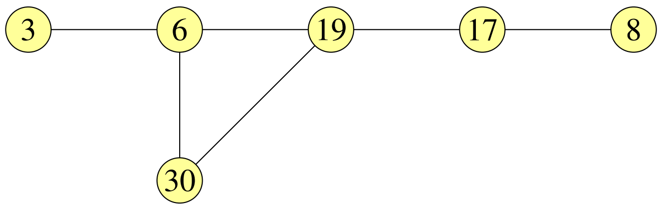
2	34, 47
3	6
6	3, 19, 30
8	17
17	8, 19, 47
19	6, 17, 30
30	6, 19, 34
34	2, 30, 47
47	2, 17, 34

From the table we see that 3 and 8 only have 1 potential neighbour. So they must be at the ends of the sequence.

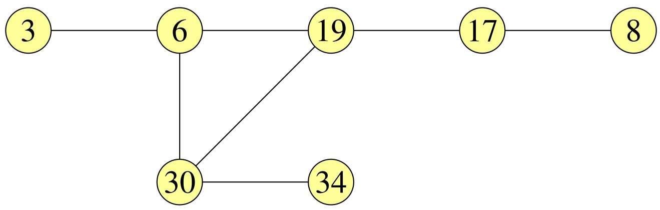
The only neighbour of 3 is 6, and the only neighbour of 8 is 17.
So the square sum sequence is 3 6 ? ? ? ? ? 17 8.

At this stage it will be convenient to draw a graph of our progress to date.

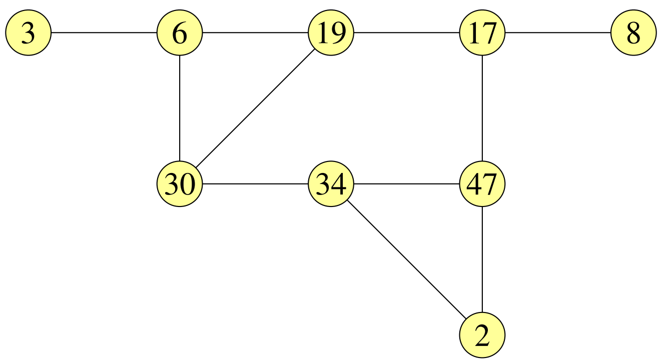
We now add the remaining potential neighbours of 6.
(We could equally have added the neighbours of 17.)



And the remaining potential neighbour of 30.



Finally the remaining potential neighbours of 34.



From the graph the square sum sequence including all of the numbers is
3 6 19 30 34 2 47 17 8.

There are 3 numbers between 19 and 47.

Hence (D).

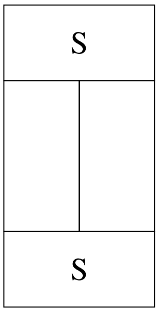
Part B: Questions 7–9

7. Plantings

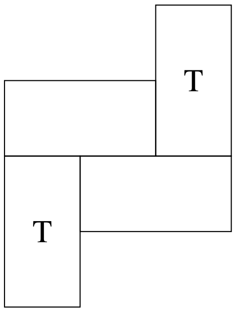
If two beds are each adjacent to two beds that are also adjacent to each other, then the two beds will be planted with the same species.

Examples:

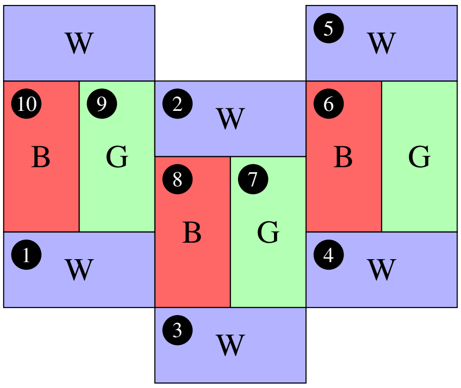
Beds S will have the same plants



Beds T will have the same plants

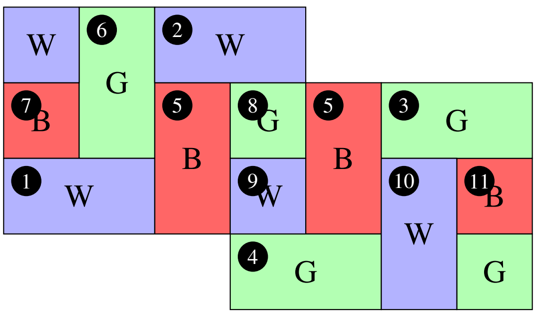


A.



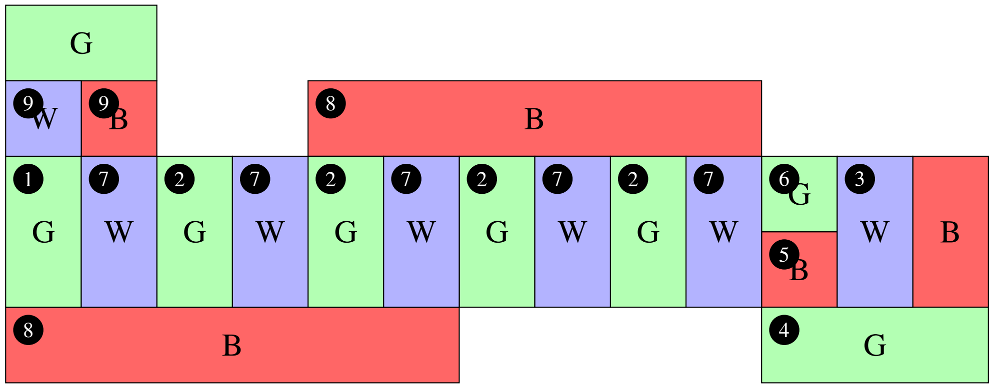
banksias: 3 beds
grevilleas: 3 beds.
waratahs: 6 beds.
Hence 336.

B.



banksias: 4 beds
grevilleas: 5 beds.
warratahs: 5 beds.
Hence 455.

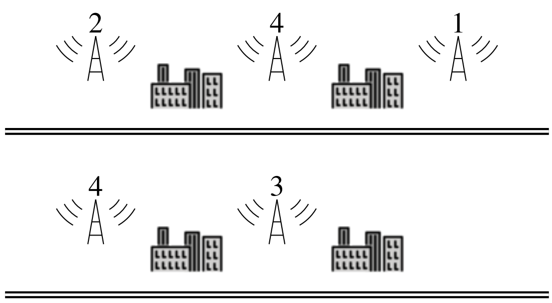
C.



banksias: 5 beds
grevilleas: 8 beds.
warratahs: 7 beds.
Hence 587.

8. Communication towers

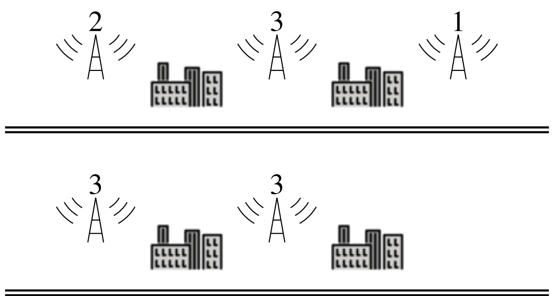
Consider maps below, where the numbers indicate the cost of building a tower on that site.



In each case the town(s) covered by the tower with cost 4 can be covered more cheaply with neighbouring tower(s). This leads to the following rule

1 If the cost of a tower at a site is greater than the cost of building towers at neighbouring site(s), it is better not to build a tower at that site.

Now consider



In the first case the towns covered by the tower with cost 3 can be covered at the same cost with neighbouring tower(s). In the second case, a tower at the leftmost site only covers one town which is covered at the same cost by a tower at the other site. And in each case the neighbouring sites cover other towns, which may lead to a cheaper overall cost. This leads to the following rule

2 If the cost of a tower at a site is greater than or equal to the cost of building towers at neighbouring site(s), it is no better to build a tower at that site.

We will use rule 2 to determine the lowest cost in our solutions.

A. 1 2 5 3 2 8 6 5 6 1

Using our rule, we build towers on the neighbours of the sites with costs 5, 8 and the second 6. This results in

1 ② 5 ③ ② 8 ⑥ ⑤ 6 ①

for a cost of $2+3+2+6+5+1 =$ 19.

```
  2       3   2       6   5       1
 ↙ ↘     ↙ ↘ ↙ ↘     ↙ ↘ ↙ ↘     ↙
T   T   T   T   T   T   T   T   T
```

B. 3 2 1 2 5 3 2 3 4 5

Using our rule, we build towers on the neighbour of each of the end sites, and the neighbours of the site in the middle with cost 5. This results in

3 ② 1 ② 5 ③ 2 3 ④ 5

The only town not covered is that between the sites costing 2 and 3. We build on the site with cost 2.

3 ② 1 ② 5 ③ ② 3 ④ 5

The cost is $2+2+3+2+4 = 13.$

```
  2       2       3   2       4
 ↙ ↘     ↙ ↘     ↙ ↘ ↙ ↘     ↙ ↘
T   T   T   T   T   T   T   T   T
```

C. 4 3 2 1 4 7 2 6 5 3 4 1

Using our rule, we build towers on the neighbour of the left end sites, and the neighbours of the site with cost 7 and the rightmost site with cost 4. This results in

4 ③ 2 1 ④ 7 ② 6 5 ③ 4 ①

There are two towns not covered, that between sites costing 2 and 1, and that between sites costing 6 and 5. We build on the sites with costs 1 and 5.

4 ③ 2 ① ④ 7 ② 6 ⑤ ③ 4 ①

The cost is $3+1+4+2+5+3+1 = 19$.

```
   3        1    4        2        5    3        1
 ↙   ↘    ↙   ↘ ↙   ↘    ↙   ↘    ↙   ↘ ↙   ↘    ↙
T     T   T     T     T  T     T  T     T     T  T
```

9. Bus passengers

We will refer to the passengers who got on at A as 'the originals', and those who got on later as 'the subsequents'.

Our aim is to have as many originals as possible on the bus at each leg. So we assume that when passengers get off, as many as possible are subsequents, keeping in mind the requirement that every passenger must complete at least one leg. We only need to keep track of the *number* of originals and the *number* of subsequents on the bus for each leg.

A.

Stop	A		1		2		3		4		5		6		B
On	20		10		0		5		0		5		0		–
Off	–		0		8		0		10		0		3		19
Originals		20		20		20		20		17 20-3		17		17	
Subsequents		0		10 0+10		2 10-8		7 2+5		0 7-7		5 0+5		2 5-3	

At most 17 passengers could have travelled from A to B.

B.

Stop	A	1	2	3	4	B
On	20	10	4	7	0	–
Off	–	0	8	10	5	18

Originals	20	20	20	16 20-(10-6)	16
Subsequents	0	10	6 (10-8)+4	7 (6-6)+7	2 (7-5)+0

At most 16 passengers could have travelled from A to B.

C.

Stop	A	1	2	3	4	5	6	B
On	20	5	6	4	8	5	5	–
Off	–	8	3	6	10	7	7	12

Originals	20	12 20-8	12	12	8 12-(10-6)	8	7 8-(7-6)
Subsequents	0	5	8 (5-3)+6	6 (8-6)+4	8 (6-6)+8	6 (8-7)+5	5 (6-6)+5

At most 7 passengers could have travelled from A to B.