Part A: Questions 1–6

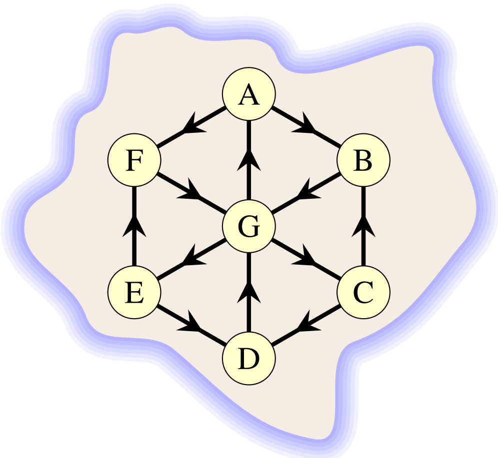
Each question should be answered by a single choice from A to E.
每题有五个选项，考生应从中选出一个正确答案。
Questions are worth 3 marks each.
每题 **3** 分。

---

1. Octave Island-奥克塔夫岛

Octave Island has seven small towns labelled A to G. The towns are connected by one-way roads as shown.

岛上有七个小镇，分别用字母 A 到 G 进行标记。各小镇之间通过单行道相连，如图所示。



Mabel lives in A and needs to visit G, E and D in order, then return to A. But she can't drive directly from A to G because of the one-way roads. Each road takes 1 hour to travel on, so her return journey takes 6 hours.

住在 A 小镇的 Mabel 想按照 G、E、D 的顺序去这三个小镇旅游，然后再回到 A 小镇。但由于各小镇之间都是单行道，她无法直接从 A 小镇去到 G 小镇。每条单行道的行驶时间为 1 小时，Mabel 往返路程一共耗时 6 小时。

Mabel's friend Klara lives in C and wants to visit the towns A, F and E in order and return to C.

Mabel 的朋友 Klara 生活在 C 小镇，想按照 A、F、E 的顺序去这三个小镇旅游，然后再回到 C 小镇。

What is the shortest time, in hours, that Klara could take for the return journey?

请问 Klara 往返路程最少需要多少个小时？

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

2. Wandering drone-无人机漫步

Mia and Adam were given a toy drone for Christmas. The remote control for this drone has five buttons.

Mia 和 Adam 在圣诞节当天收到了一架玩具无人机。这架无人机可以通过五个按钮进行远程操控。

U : Go up in the air 10 metres 上升 10 米

D : Go down towards the ground 10 metres 下降 10 米

L : Turn left 90° 左转 90°

R : Turn right 90° 右转 90°

F : Go forward 1 metre 前进 1 米

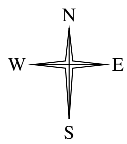
They took it to the local park and set it down on the ground facing north.

Mia 和 Adam 把无人机面朝北放在公园地上。

They then pressed the following buttons in order.

然后按顺序执行下列按钮指令。

U F R R F F L R R L F L F L F L F F R L L R F F R R F R F D



What direction is the drone facing now?

请问无人机现在面对哪个方向？

(A) north 北 (B) east 东 (C) south 南

(D) west 西 (E) it’s impossible to tell 无法判断

3. Growth-野蛮生长

The game of *Growth* takes place on a grid of white squares.

野蛮生长 游戏采用一个由白色方格组成的网格。

- To start, a single square somewhere in the grid is chosen. It is shaded.

  游戏开始时，网格中随机出现一个阴影方格。

- To make a move, a player chooses a shaded square and shades two white squares that are adjacent to it. (Adjacent means the squares have an edge in common.)

  玩家每一轮需要选择一个阴影方格，然后为其两个相邻白色方格涂上阴影。(相邻方格指的是共用一条边的方格。)

- A shaded square **cannot** be chosen if it does not have two white squares adjacent to it.

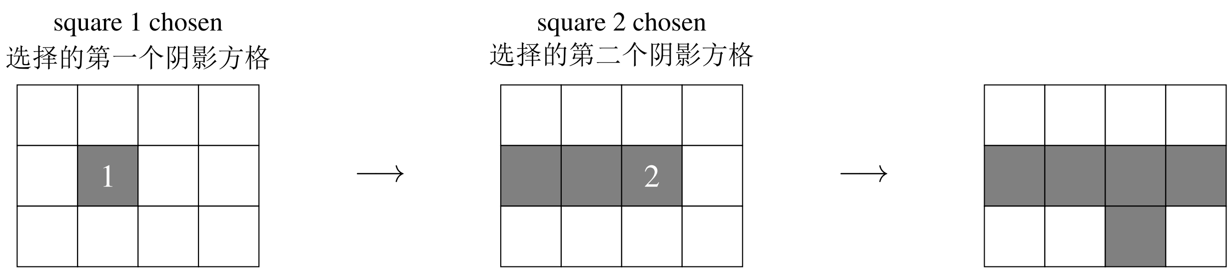
  如果一个阴影方格没有两个相邻的白色方格，那么它就**不能**被选择。

- A shaded square **can** be chosen more than once.

  一个阴影方格**可以**被多次选择。

The example below shows the first shaded square and how the board *could* develop over two moves on a 4×3 grid.

下图显示了在这个 4×3 网格中选择的第一个阴影方格以及经过两轮操作后网格可能的变化情况。

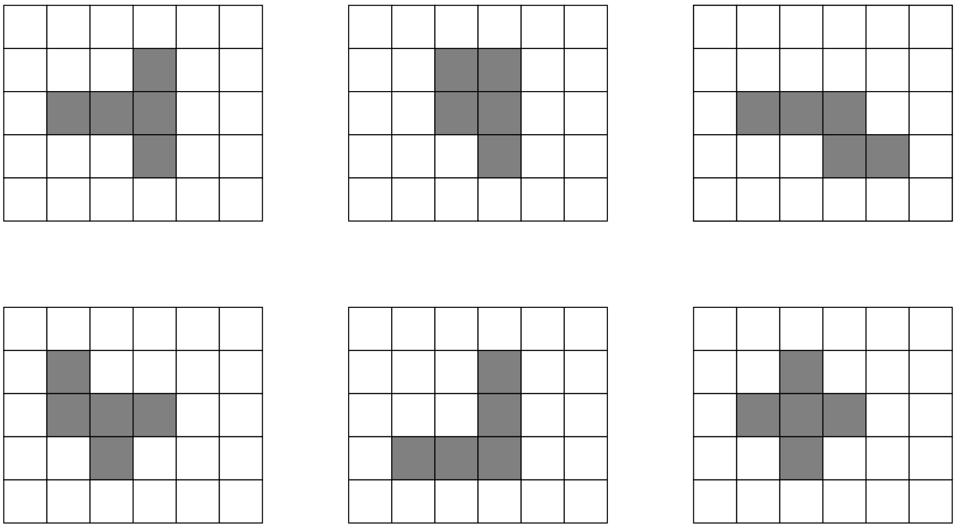


A game is played on a 6×5 grid.

如果该游戏采用的是一个 6×5 的网格。

How many of the following diagrams are possible after two moves?

请问在下列图片中，有多少张图片可能显示的是两轮操作之后的结果？



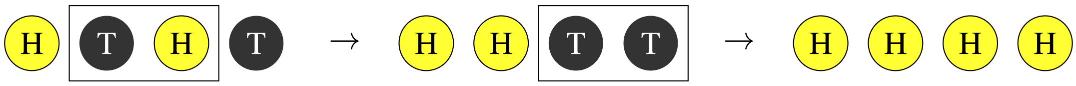
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

4. Heads up-翻面

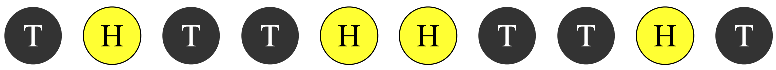
You have a line of coins, some with the head side up and some with the tail side up. You want all of the coins to have the head side up. The only move you are allowed is to flip two adjacent coins. You will apply this move repeatedly until all the coins have their head sides up.

一排硬币中，有些正面（H）朝上，有些背面（T）朝上。此时，你想让一排硬币全部正面（H）朝上。操作过程中，每轮只能同时翻转两个相邻硬币，一直翻转直到所有硬币都正面（H）朝上为止。

Example 举例



You are given the following line:
给定如下一排硬币：



What is the smallest number of moves you need so that all the coins have their head sides up?

请问至少需要翻转几轮才能使所有硬币正面（H）朝上？

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

5. Garry's moves-Garry 的棋局

Garry is playing a game on an 8 by 8 chessboard. Here are the rules:

Garry 正在一个 8×8 的棋牌上玩游戏。游戏规则如下：

- On each turn he can move his playing piece up, down, left or right.

  每一轮操作，Garry 可以在上、下、左、右四个方向上移动自己的棋子。

  Diagonal moves are *not* allowed.

  但 禁 止 在对角线方向上移动。

- If his piece is on a dark square, on his next turn he must move exactly 1 square.

  如果棋子落在黑色方格内，下一轮操作时必须恰好移动一个方格。

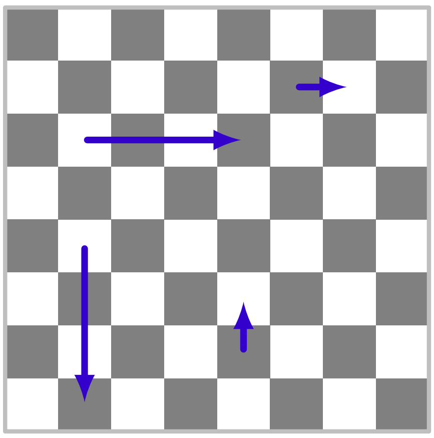
- If his piece is on a white square, on his next turn he must move exactly 3 squares, all in the same direction.

  如果棋子落在白色方格内，下一轮操作时就必须在同一方向上恰好移动 3 个方格。

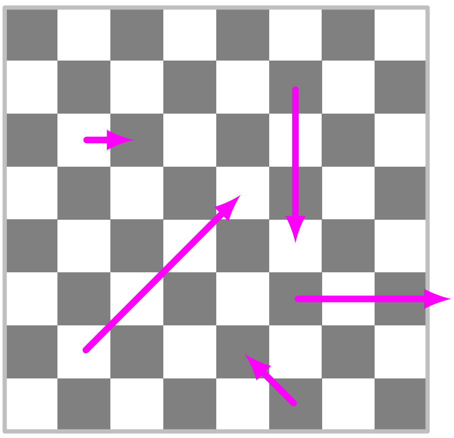
---

*Examples*举例：

Allowed 允许

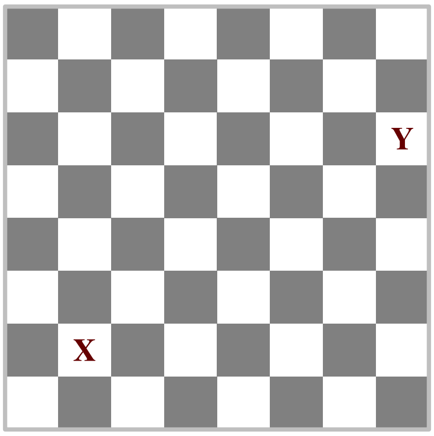


Not allowed 禁止



On the chessboard below, Garry wants to get his piece from **X** to **Y** in the smallest possible number of turns.

在下图所示棋盘中， Garry 想通过尽可能少的操作次数将自己的棋子从 **X** 方格移动到 **Y** 方格。



How many turns will he take?

请问他最少需要经过几轮才能将棋子从 **X** 方格移动到 **Y** 方格？

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

6. Pancakes-煎饼

There are three plates in front of you: two sorting plates (grey) and one serving plate (orange).

面前有三个盘子：两个灰色的分拣盘和一个橘色的餐盘。

You can move pancakes between the sorting plates *however you wish*, but:

你可以 按照自己的喜好，在两个分拣盘之间移动煎饼，但是：

- you can only move a pancake from the top of one pile to the top of another pile (or to an empty plate); **and**

  只能将其中一个分拣盘最上方的煎饼移到另一个分拣盘的最上方（或者移到一个空盘子里）；**并且**

- once you move a pancake to the serving plate, *you cannot move it again*.

  一旦煎饼被放到餐盘中，就不能再次移动该煎饼。

---

*Example*举例:

You can move the 4-pancake to the other sorting plate or to the serving plate.

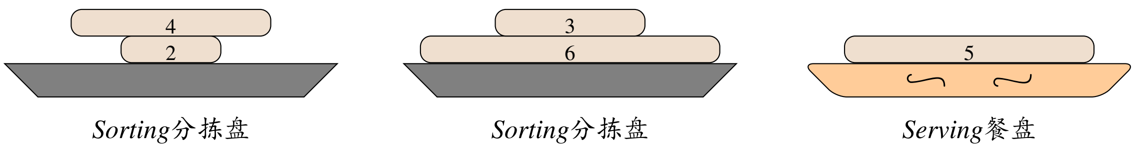
你可以将 4 号煎饼移动到另一个分拣盘或者餐盘中。

You can move the 3-pancake to the other sorting plate or to the serving plate.

你可以将 3 号煎饼移动到另一个分拣盘或者餐盘中。

You cannot move the 5-pancake since it is on the serving plate.

不能移动 5 号煎饼，因为 5 号煎饼已经放在餐盘中了。



At the start, the leftmost sorting plate contains a pile of pancakes in a random size order. The other two plates are empty.

游戏开始时，最左边的分拣盘中摆放着这一叠煎饼，这些煎饼并未按照尺寸大小顺序排列。与此同时，其他两个盘子为空。

Your goal is to move all pancakes to the serving plate, in sorted order, with the smallest on the top and the largest on the bottom.

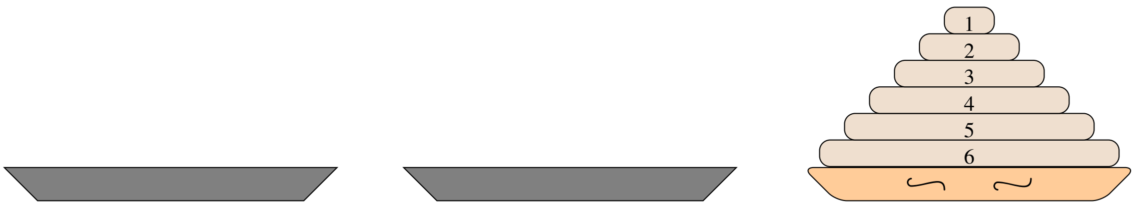
你需要将所有煎饼移动到餐盘中，并按数字顺序摆放：数字最大的煎饼放在最下面；数字最小的煎饼放在最上面。

Initial: All pancakes on the left plate, in order 4 2 5 3 6 1 (with 4 on the top).

游戏开始时：所有煎饼按照 4 2 5 3 6 1 的顺序摆放在左边的分拣盘中（4 号煎饼摆放在最上面）。

Final: All pancakes on the serving plate, in order 1 2 3 4 5 6 (with 1 on the top).

游戏结束时：所有煎饼按照 1 2 3 4 5 6 的顺序摆放在餐盘中（1 号煎饼摆放在最上面）。



How many moves would it take to arrange them in sorted order on the serving plate, with the smallest on the top and the largest on the bottom?

请问要经过多少次移动才能将所有煎饼按顺序放在餐盘中：数字最小的煎饼放在最上面，数字最大的煎饼放在最下面？

(A) 9 (B) 10 (C) 11 (D) 13 (E) 15

Part B: Questions 7–9

Each question has three parts, each of which is worth 2 marks.
每题有三个部分，每部分 2 分。
Each part should be answered by a number in the range 0–999.
每部分答案应为一个介于 0–999 之间的数字。

---

7. Restricted swapping-限制交换

You are given a number with several digits. You are allowed to make several swaps. Each swap is of two adjacent digits. Your aim is to form the largest number possible.

给定一个多位数。玩家需要对这个多位数的各位数字进行多次顺序交换，最终得到一个可能的最大数字。玩家每次只能对两位相邻数字进行交换。

For example, suppose you start with 8 5 3 7 9 2 and are allowed to make three swaps. You might choose

假设给定的多位数是 8 5 3 7 9 2，且玩家仅允许进行 3 次交换。你可以选择

8 <u>5 3</u> 7 9 2 → 8 3 5 7 <u>9 2</u> → 8 3 5 <u>7 2</u> 9 → 8 3 5 2 7 9

Note that with these choices you have made the number *smaller*, not larger!

注意，以上三次交换是让数字变小了，而不是变大!

You are presented with 2 9 1 7 8 6 5.

现在给定一个多位数 2 9 1 7 8 6 5。

What are the **middle three digits** of the largest number you can make with:

请问对于以下三种不同的交换次数，分别得到的最大数字的**中间三位数字**是多少？

A. 3 swaps? 3 次交换？

B. 4 swaps? 4 次交换？

C. 7 swaps? 7 次交换？

For instance, if the largest number you can make is 1 2 6 9 7 5 8, your answer will be 6 9 7.

例如，如果得到的最大数字是 1 2 6 9 7 5 8，那么你的答案应该为 6 9 7。

8. Plantings-苗床种植

Elle wants to plant her rural block with banksias (B), grevilleas (G) and waratahs (W). She has marked out several rectangular garden beds. She wants to plant out her garden beds so that:

Elle 想要在农村种植山茂樫（B）、银桦木（G）、瓦松（W）。她已经将土地规划成多块矩形苗床，并且计划按照以下模式在苗床中栽种：

- each garden bed will be planted with only one type of plant – banksias or grevilleas or waratahs

  每块苗床只栽种一种植物——山茂樫（B）、银桦木（G）或瓦松（W）

- neighbouring garden beds will not be planted with the same type of plant (neighbouring garden beds are those that have an edge or part of an edge in common, but not just a corner).

  相邻苗床中栽种的植物类型不同（相邻苗床指的是共用一条边或者共用一部分边的苗床。注意，仅共用一个角落的苗床不是相邻苗床。）

Just as she is pondering the possibilities, her mother announces that she has just put some plants in! Luckily the rules Elle set have been obeyed, and now there is no decision to make: there is only one possible way to plant the rest of the beds.

正当 Elle 考虑计划可行性时，她的妈妈说她已经在一些苗床里面种了一些植物了！不过，幸好妈妈遵循了 Elle 设定的栽种规则，现在 Elle 只能在剩下未种植的苗床中栽种植物了。

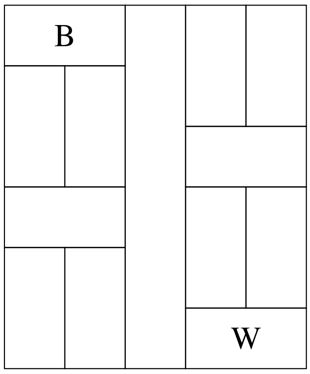
For each of the rural blocks below, find the number of beds that are planted with each type of plant. *This includes the beds Elle's mother has planted.*

对于下列各块农村土地，求每种植物的苗床数量。其中包含 *Elle* 妈妈栽种的苗床。

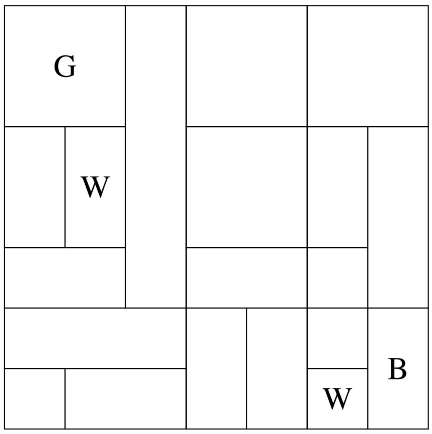
Your answer will be a 3-digit number, giving the number of beds planted with banksias, grevilleas and waratahs. For instance 123 would mean 1 bed planted with banksias, 2 with grevilleas and 3 with waratahs.

你的答案应该为一个 3位数，分别表示山茂樫苗床、银桦木苗床、瓦松苗床的数量。例如数字123表示 1 个山茂樫苗床、2 个银桦木苗床和 3 个瓦松苗床。

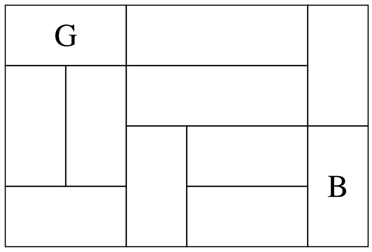
A.



B.



C.



9. Sami the sales rep-销冠 Sami

Sami has clients in several towns along a road.

Sami 的客户分布在沿路的几个小镇中。

On her outward journey her expenses are covered and she can keep all the money she makes from sales. On her return home, however, she must pay her travel expenses.

外出拜访客户时，Sami 的花费都可以报销，因此她的销售收入可以完全存下来。但在返程途中，她只能自己支付路费。

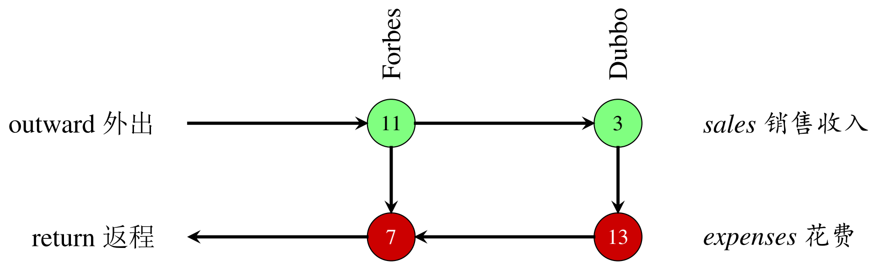
Sami does not have to visit all of the towns. She can choose where she turns back.

Sami 可以选择从任意一个小镇返程，不必去到所有小镇。

Example 举例:

Along this road, if she turns back at Forbes she makes \$11 in sales but it costs her \$7 in expenses.
如果她从 Forbes 返程，她的销售收入为 \$11 ，但需要自行支付路费 \$7 。

If she goes on to Dubbo she makes $\$11+\$3=\$14$ in sales but it costs her $\$7+\$13=\$20$ in expenses.
如果她从 Dubbo 返程，她的销售收入为 $\$11+\$3=\$14$ ，但需要自行支付路费 $\$7+\$13=\$20$ 。



In this case there is only one travel option where her sales are greater than her expenses – turning back at Forbes.

在这种情况下，Sami 只有从 Forbes 返程，才能保证销售收入比路费多。

For each of the following roads, how many travel options are there where her sales are **greater than** her expenses?
请问在下列三条路线中，分别有多少种返程点的选择能使 Sami 的销售收入比路费 多?

A.

	A	B	C	D	E	F
outward 外出	8	7	4	6	3	5
return 返程	5	8	7	4	1	8

B.

	A	B	C	D	E	F	G	H	I
outward 外出	9	6	5	6	5	3	5	4	6
return 返程	8	8	3	7	6	1	4	7	4

C.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
outward 外出	5	4	2	6	5	3	3	7	5	1	4	2	3	6
return 返程	6	3	1	5	6	6	5	2	6	0	3	3	5	4

Solutions

Part A: Questions 1–6

---

1. Octave Island

Travelling between the towns on the list may require going via other towns.

(B|D) means that Kara could go via B or D.

Towns		Hours
leg	route	
C → A	C → (B|D) → G → A	3
A → F	A → F	1
F → E	F → G → E	2
E → C	E → (F|D) → G → A	3

Kara's journey will take a total of $3+1+2+3=9$ hours.

Hence (D).

2. Wandering drone

The Us, Ds and Fs do not affect the way the drone is facing, so can be ignored.

This leaves 9 Rs and 7 Ls. As an R and an L cancel each other out, this is equivalent to 2 Rs.

The first R turns the drone east, and the second R turns it south.

Hence (C).

3. Growth

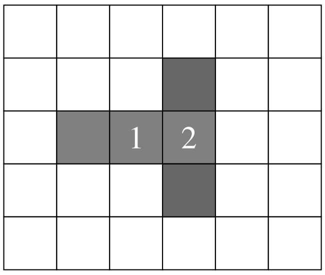
We make the following observations: 观察如下：

1. If there is exactly one square with 2 neighbours, it must have been the first square chosen.
2. If there is more than one square with 2 neighbours, the final diagram is not possible.

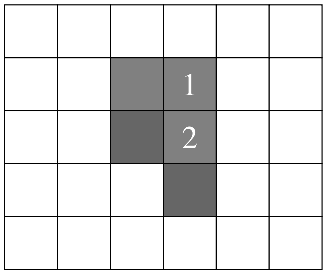
We use these rules to determine the order in which squares were selected.

Diagrams (A), (B), (D) and (F) could be the board position after two moves;
Diagrams (C) and (E) would require choosing two squares that are not adjacent.

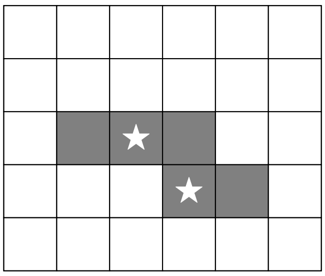
(A)



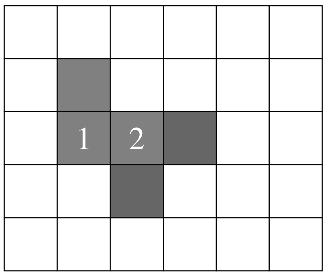
(B)



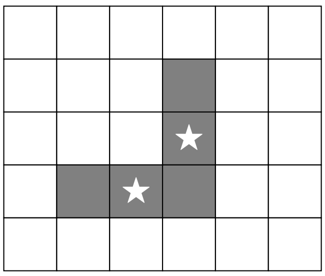
(C)



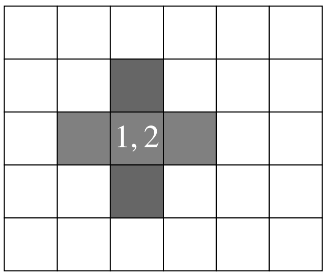
(D)



(E)



(F)



Hence (E).

4. Heads up

Solution 1

We make the following observations:

1 The options for flipping two coins are

$$\begin{array}{ccc} \mathrm{T\,T} & \rightarrow & \mathrm{H\,H} \\ \mathrm{T\,H} & \rightarrow & \mathrm{H\,T} \\ \mathrm{H\,T} & \rightarrow & \mathrm{T\,H} \\ \mathrm{H\,H} & \rightarrow & \mathrm{T\,T} \end{array}$$

In each case if you start with an even number of Ts you end with an even number of Ts. So if there was an even number of Ts in the line, there will always be an even number of Ts in the line and it will be possible to end up with no Ts.
(This is the case for the line in the question.)

2 If a coin is flipped with both of its neighbours, it does not matter in which order the flips are executed.

Observation 2 enables us to develop a left-to-right algorithm.
If the first coin is T, we flip the first two coins.
Then if the second coin is T, we flip the first and second coins, and so on.

So in this case the first coin is a T so the first two coins must be flipped.
This flips the second coin from an H to a T, so the second and third coins must be flipped.
This also flips the third coin from a T to an H, so the third and fourth coins do not need to be flipped.

In the diagram below, pairs of coins that need to be flipped are indicated by a $\frown$.

$$\mathrm{T} \frown \mathrm{H} \frown \mathrm{T} \quad \mathrm{T} \frown \mathrm{H} \frown \mathrm{H} \frown \mathrm{T} \quad \mathrm{T} \frown \mathrm{H} \frown \mathrm{T}$$

Seven flips are required.

Hence (D).

Solution 2

Here we make a further observation

3 Consider the sequence of coins T H ... H T with n Hs.
This can be be changed into H H ... H H with $(n+1)$ flips.

$\underline{\mathrm{T\,H}}\,\mathrm{H\,T} \rightarrow \mathrm{H}\,\underline{\mathrm{T\,H}}\,\mathrm{T} \rightarrow \mathrm{H\,H}\,\underline{\mathrm{T\,T}} \rightarrow \mathrm{H\,H\,H\,H}$

We can use this observation to deduce the number of flips required without tracing through the algorithm.

$\mathrm{THTTHHTTHT} = \mathrm{THT\ \ THHT\ \ THT}$

T H T	requires 2 flips
T H H T	requires 3 flips
T H T	requires 2 flips

Seven flips are required.

Hence (D).

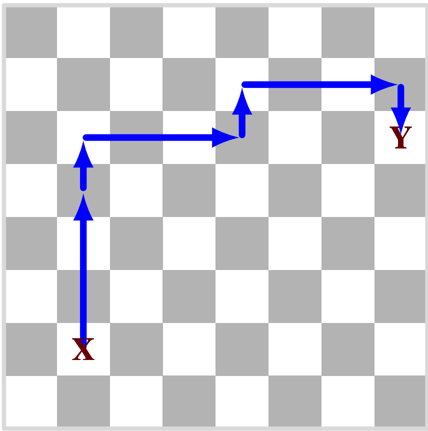
5. Garry's moves

Whether he moves 1 or 3 steps, the colour of the square he is on will change from dark to white or vice versa. Since his first move is from a white square, the numbers of steps on each turn will alternate 3, 1, 3, 1 and so on.

The shortest one-square-at-a-time path from **X** to **Y** is 10 steps (for example, 6 to the right then 4 up). But using 4 turns will take him at most $3+1+3+1=8$ steps away from **X** which is not enough. So he will need more than 4 turns.

Using 5 turns will take him $3+1+3+1+3=11$ steps away from **X**. But dark and white squares alternate, so any such path will end on a dark square, not at **Y**. So he will need more than 5 turns. (This also rules out 7 turns.)

The diagram shows one way to get from **X** to **Y** in 6 turns:



Hence (C).

6. Pancakes

Before moving pancake 6 to the third plate, the pancakes above it have to be moved to the second plate. Then the pancake above pancake 5 has to be moved to the first plate, and so on.

4		
2		
5		
3		
6		
1	–	–

$\xrightarrow{5\ moves}$

	3	
	5	
	2	
1	4	6

$\xrightarrow{2\ moves}$

3	2	5
1	4	6

$\xrightarrow{2\ moves}$

2		4
3		5
1	–	6

$\xrightarrow{2\ moves}$

		3
		4
		5
1	2	6

$\xrightarrow{2\ moves}$

		1
		2
		3
		4
		5
–	–	6

A total of $5+2+2+2+2=13$ moves are required.

Hence (D).

Part B: Questions 7–9

7. Restricted swapping

Your aim is always to move as large a digit as possible to the left of the list.

A. 2917865: 3 swaps

Your first swap is to move the 9 to the left of the list.

2917865 → 9217865

You have 2 swaps left.

The 8 can't be moved next to the 9 in 2 steps, so you have to use the 2 steps to move 7 next to the 9.

9217865 → 9271865 → 9721865

The largest number you can form is 9721865

Hence 218.

B. 2917865: 4 swaps

Your first swap is to move the 9 to the left of the list.

2917865 → 9217865

You have 3 swaps left.

You will use them to move the 8 next to the 9.

9217865 → 9218765 → 9281765 → 9821765

The largest number you can form is 9821765

Hence 217.

C. 2917865: 7 swaps

Your first swap is to move the 9 to the left of the list.

2917865 → 9217865

You have 6 swaps left.

You will use 3 of them to move the 8 next to the 9. *(This is the same as part B.)*

9217865 → 9218765 → 9281765 → 9821765

You will use 2 of them to move the 7 next to the 8.

9821765 → 9827165 → 9872165

You have 1 swap left.

The best you can do is to advance the 6.

9872165 → 9872615

The largest number you can form is 9872615

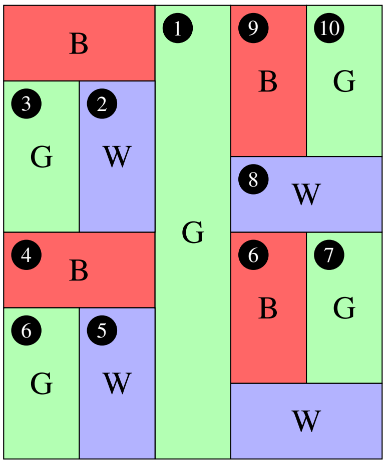
Hence 726.

8. Plantings

If a bed has two adjacent beds that have been planted with different species it will be planted with the third species.

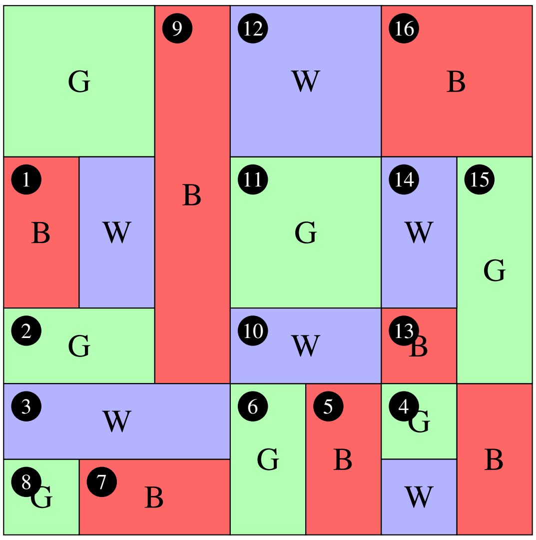
In the diagrams below the number in the circle represents one order in which the beds could be planted.

A.



4 beds have been planted with banksias.
5 beds have been planted with grevilleas.
4 beds have been planted with waratahs.
Hence 454.

B.

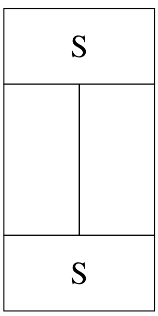


7 beds with banksias.
7 beds with grevilleas.
6 beds with waratahs.
Hence 776.

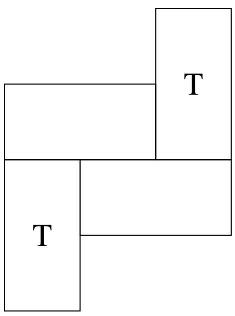
If two beds are each adjacent to two beds that are also adjacent to each other, then the two beds will be planted with the same species.

Examples:

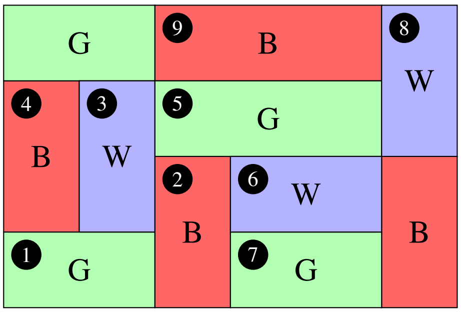
Beds S will have the same plants



Beds T will have the same plants



C.



4 beds with banksias.
4 beds with grevilleas.
3 beds with waratahs.
Hence 443.

9. Sami the sales rep

We calculate the total sales to each town and the total expenses from each town.

A.

	A	B	C	D	E	F
outward	8	7	4	6	3	5
sales	8	15	19	25	28	33
return	5	8	7	4	1	8
expenses	5	13	20	24	25	33
sales > expenses	✓	✓	✗	✓	✓	–

Returning at four cities would be profitable.

Hence 4.

B.

	A	B	C	D	E	F	G	H	I
outward	9	6	5	6	5	3	5	4	6
sales	9	15	20	26	31	34	39	43	49
return	8	8	3	7	6	1	4	7	4
expenses	8	16	19	26	32	33	37	44	48
sales > expenses	✓	✗	✓	–	✗	✓	✓	✗	✓

Returning at five cities would be profitable.

Hence 5.

C.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
outward	5	4	2	6	5	3	3	7	5	1	4	2	3	6
sales	5	9	11	17	22	25	28	35	40	41	45	47	50	56
return	6	3	1	5	6	6	5	2	6	0	3	3	5	4
expenses	6	9	10	15	21	27	32	34	40	40	43	46	51	55
sales >	✗	–	✓	✓	✓	✗	✗	✓	–	✓	✓	✓	✗	✓

Returning at eight cities would be profitable.

Hence 8.