

Grade 3-4

At most how many students can sit in a row of 25 chairs, if seated students must be separated by at least two empty chairs?



- a. 9
- b. 10
- c. 11
- d. 12
- e. 13
- f. 14
- g. 15

如果一排有 25 把椅子，学生与学生之间必须隔有至少两个空椅子，请问最多有多少学生可以坐在这排里？



- a. 9
- b. 10
- c. 11
- d. 12
- e. 13
- f. 14
- g. 15

Solution

To maximize the number of students that can sit in these chairs, it is useful to think where the leftmost student should sit. Having a student sit on the leftmost chair is definitely not worse than having him/her sitting on any other chair because sitting on the very left means that maximal space is available on the right of that student. Having one or two free seats to the left of the student does not allow another student to sit on the left but may reduce the number of students sitting on the right.

Because of the left \leftrightarrow right symmetry of the question, we equally could start by placing a student on the rightmost chair and get the same solution.

1. Solution

So, when the first person sits in the leftmost chair, the next person should sit in the 4th chair (because we should leave at least 2 chairs empty) and the other students would sit on the 7th, 10th, 13th, 16th, 19th, 22nd, 25th chairs. In total 9 students will get a seat.

2. Solution

If the number c of chairs is large then counting the occupied ones would be too much effort. We can compute that number s of students on chairs by

- subtracting 1 from c for the leftmost chair which will be occupied,
- dividing the remaining number $c-1$ of chairs by 3 (2 free chairs + the next occupied chair = 3 chairs), and
- adding 1 to it for the leftmost student.

In the case of this question we get

$$(25-1)/3+1 = 24/3+1 = 8+1 = 9.$$

If $c-1$ is not divisible by 3 then we only take the integer part from this division. For example, if there would be 27 chairs then $(27-1)/3 = 8$ remainder 2 so we would use 8 and by adding 1 for the leftmost student, we would still get only 9 occupied seats which is correct.

要使能坐在这排的学生最多，首先要考虑最左边的学生应该坐在哪里。显然，让其坐最左边的椅子比坐其他椅子要好，因为坐最左边椅子时其右边能坐的学生最多，而如果在其左边空一两把椅子，其他学生也不能坐从而减少了右边可以坐的学生数量。因为左右具有对称性，同理我们也可以考虑首先让一名学生坐最右边的椅子，结论相同。

解法 1：当第一个人坐最左边的椅子，第二个人则应该坐第四把椅子（因为我们要留至少 2 把空椅子），其余的人分别可以坐在第 7 把，第 10 把，第 13 把，第 16 把，第 19 把，第 22 把，第 25 把椅子上，则这一排总共可以坐 9 名学生。

解法 2：当椅子的数量 c 过多时，上面计数的方法就太费力了，我们可以通过以下方法计算出可以坐的学生数量 s

从 c 把椅子中抽出一把给最左边的人坐

将剩余的椅子数量 $c-1$ 除以 3（2 把空椅子加上下一把被坐的椅子，共计 3 把），再加上留给最左边学生的一把，在本题中即可得出可以坐的椅子数量为：

$$(25-1)/3+1 = 24/3+1 = 8+1 = 9$$

如果 $c-1$ 不能被 3 整除，我们只取整数部分。例如，如果共有 27 把椅子，则 $(27-1)/3 = 8$ 还余 2，则我们可以坐的椅子数量为 8 加上留给最左边学生的 1 把，共 9 把。

How many cats should be drawn on the right hand side of '=' to make it a correct statement?



为了使下面的描述正确，我们应该在 '=' 右边画几只小猫？



- a. 3
- b. 4
- c. 5
- d. 6
- e. 1
- f. 2
- g. 7

Solution

The '+' sign means that we need to add the two numbers of cats. We need to add 2 and 3 giving 5. We also could simply count all cats on the left and would get again 5.

加号“+”表示我们需要把两组小猫的数量加起来，也就是 2 加上 3，可以得到共有 5 只小猫。我们也可以直接数一数发现左边小猫共有 5 只。

Grade 5-6

A square is NOT

- a. a special type of parallelogram
- b. a special type of polygon
- c. a special type of rectangle
- d. made from equal length sides
- e. **made from sides of different length**
- f. 4-sided
- g. composed of 90° angles

正方形不是

- a. 一种特殊的平行四边形
- b. 一种特殊的多边形
- c. 一种特殊的长方形
- d. 由等长的边组成
- e. **由不等长的边组成**
- f. 4条边
- g. 由 90° 度角组成

Solution

A square is a special type of parallelogram, polygon, rectangle and it has 4 equally long sides and four 90° angles. Therefore, it is not made from sides of different lengths.

正方形是一种特殊的平行四边形，多边形，长方形，它有4个等长边和4个 90° 角。因此，它不是由不等长边组成的。

At the Yellow Star Taxi company Sandra is a receptionist and Dave is a driver. *Not including Sandra* there are 4 receptionists. *Not including Dave*, there are 21 drivers employed. How many receptionists and drivers in total are employed by the taxi company?

- a. 5
- b. 21
- c. 22
- d. 25
- e. 26
- f. 27
- g. 28

在黄星出租车公司，桑德拉是一名接待员，戴夫是一名司机。该公司不包括桑德拉在内共有4名接待员，不包括戴夫在内共有21名司机。该出租车公司共有多少名接待员和司机？

- a. 5
- b. 21
- c. 22
- d. 25
- e. 26
- f. 27
- g. 28

Solution

Not including Sandra, there are 4 receptionists \Rightarrow there are $4+1 = 5$ receptionists employed.

Not including Dave, there are 21 drivers \Rightarrow there are $21+1 = 22$ drivers employed.

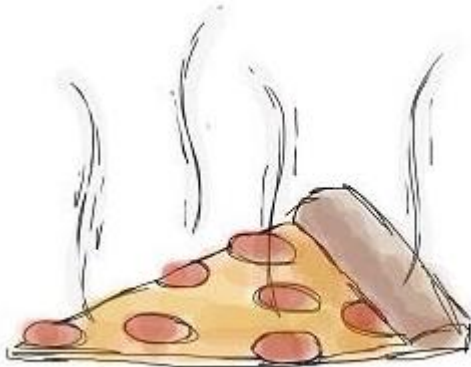
To get the total number of employed receptionists and drivers we have to add $5 + 22 = 27$.

Solution

不包含桑德拉在内有 4 名接待员，则公司共有 $4+1 = 5$ 名接待员
不包含戴夫在内有 21 名司机，则公司共有 $21+1 = 22$ 名司机
公司的接待员和司机共有 $5 + 22 = 27$ 名

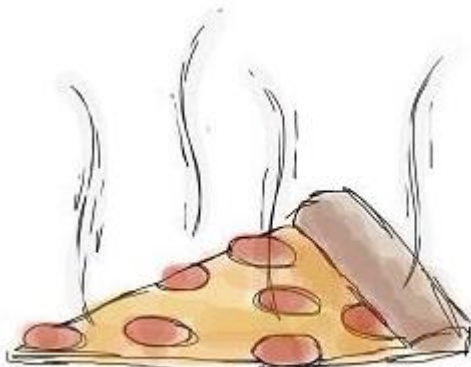
Grade 7-8

A circular pizza has an area of 900cm^2 . If a slice is cut with an angle of 60° at the centre, what is the area of the remaining pizza?



- a. 150cm^2
- b. 60cm^2
- c. 600cm^2
- d. 300cm^2
- e. 450cm^2
- f. 750cm^2
- g. 15cm^2

一个圆形披萨的面积为900平方厘米。如果切下与中心成 60° 角的部分，剩下的披萨面积是多少？



- a. 150cm^2
- b. 60cm^2
- c. 600cm^2
- d. 300cm^2
- e. 450cm^2
- f. 750cm^2
- g. 15cm^2

Solution

With a full circle having a 360° angle, the cut out slice has an area of $900\text{cm}^2 \times (60^\circ/360^\circ) = 900\text{cm}^2/6 = 150\text{cm}^2$.

Thus, the remaining pizza has an area of $900\text{cm}^2 - 150\text{cm}^2 = 750\text{cm}^2$.

一个完整的圆形披萨是 360° ，则切走部分的面积为 $900 \times (60^\circ/360^\circ) = 900/6 = 150$ 平方厘米。因此，剩余披萨的面积为 $900-150=750$ 平方厘米。

Steve plants twelve trees every thirty minutes. If he continues planting at the same rate, how long will it take him to plant 240 trees?

- a. 1 h
- b. 3 h
- c. 5 h
- d. 8 h
- e. 10 h
- f. 12 h
- g. 14 h

史蒂夫每三十分种种十二棵树，如果他继续以相同的速度种植，种植240棵树需要多长时间？

- a. 1小时
- b. 3小时
- c. 5小时
- d. 8小时
- e. 10小时
- f. 12小时
- g. 14小时

Solution

1. Solution

Steve plants twelve trees every thirty minutes. Let's find how much time he needs for 240 trees.

Because $240 / 12 = 20$, he needs 20×30 minutes = 600 minutes.

So, in order to plant 240 trees, Steve will need 600 minutes = 10 hours.

2. Solution

If it takes 30 min = $1/2$ hour to plant 12 trees then in one hour 24 trees can be planted so it takes 10 hours to plant $10 \times 24 = 240$ trees.

解答1：史蒂夫每三十分种种12棵树；又因为 $240/12=20$ ，那么他种240棵树需要 20×30 分钟=600分钟，即10小时。

解答2：如果种十二棵树需要花费30分钟即0.5小时，那么1小时则可以种24棵树。则他需要10小时种 $10 \times 24 = 240$ 棵树。

Grade 9-10

Find a pattern in this sequence: 12, -16, 21, -27, 34,
What is the next number?

- a. 40
- b. -40
- c. 41
- d. -41
- e. 42
- f. -42
- g. 43

发现规律：12, -16, 21, -27, 34, ……，下一个数是什么？

- a. 40
- b. -40
- c. 41
- d. -41
- e. 42
- f. -42
- g. 43

Solution

The rule of the pattern can be seen when think about the absolute value of these numbers \Rightarrow 12, 16, 21, 27, 34...

They are increasing consecutively; +4, +5, +6, +7...

That is, $12 + 4 = 16$, $16 + 5 = 21$, $21 + 6 = 27$, $27 + 7 = 34$...

So, the answer will be obtained by adding 8 to the last term $\Rightarrow 34 + 8 = 42$. However; because original form of the numbers in the pattern continue as one positive one negative by turns (12, -16, 21, -27, 34), the next number must be negative, the answer is -42.

首先考虑这些数的绝对值 \Rightarrow 12, 16, 21, 27, 34，可以发现他们按照如下规律依次递增：

+4, +5, +6, +7……，即 $12 + 4 = 16$, $16 + 5 = 21$, $21 + 6 = 27$, $27 + 7 = 34$ ……

因此，最后一项应该是前一项加上 8 $\Rightarrow 34 + 8 = 42$ ，但是原始序列数是一正一负的顺序（12, -16, 21, -27, 34），则最后一项应是负数，答案为 -42

Emma and Isabelle are walking side by side in the garden. They walk at the same speed but for every five steps Emma takes, Isabelle crosses the same distance in six steps. If each one of Emma's steps is 0.4 meters long, how long is the distance in metres that Isabelle walks in 30 steps?

- a. 7
- b. 8
- c. 9
- d. 10
- e. 11
- f. 12
- g. 13

艾玛和伊莎贝尔在花园里并排散步。他们以相同的速度行走，但是艾玛每走5步，伊莎贝尔要走6步才能走过相同的路程。如果艾玛每一步走0.4米，那么伊莎贝尔走30步能走多少米？

- a. 7
- b. 8
- c. 9
- d. 10
- e. 11
- f. 12
- g. 13

Solution

Let us calculate first how many times Isabelle takes six steps because we know that Emma takes 5 steps equally as often.

$30/6 = 5 \Rightarrow$ Isabelle takes 5 times 6 steps. Hence, Emma takes 5 times 5 steps = 25 steps each being 0.4 meters long, so she walks a total distance of $25 \times 0.4 \text{ m} = 25 \times 4 \text{ m} / 10 = 100 \text{ m} / 10 = 10 \text{ m}$. As both walk at the same speed Isabelle also walks 10 m while doing 30 steps.

已知伊莎贝尔每走6步艾玛则走5步，首先计算伊莎贝尔要走几个6步。
 $30/6 = 5 \Rightarrow$ 伊莎贝尔要走5个6步，因此相同时间内艾玛要走5个5步即25步，每一步0.4米，则一共能走 $25 \times 0.4 \text{ m} = 25 \times 4 \text{ m} / 10 = 100 \text{ m} / 10 = 10 \text{ m}$ 。又因为两人以同样的速度行走，则伊莎贝尔走30步也能走10m

Grade 11-12

From the eight numbers 17, 13, 5, 10, 14, 9, 12, 16 remove two numbers so that the average of the remaining six numbers is equal to the average of the original eight numbers.

- a. 5 and 17
- b. 9 and 16
- c. 10 and 12
- d. 9 and 13
- e. 5 and 16
- f. 12 and 14
- g. 10 and 14

从 17, 13, 5, 10, 14, 9, 12, 16 这八个数中移除 2 个数，使得剩下的 6 个数的平均值等于之前 8 个数的平均值。

- a. 5和17
- b. 9和16
- c. 10和12
- d. 9和13
- e. 5和16
- f. 12和14
- g. 10和14

Solution

Let us first find the average of the 8 numbers:

$$17 + 13 + 5 + 10 + 14 + 9 + 12 + 16 = 96 \text{ and } 96 / 8 = 12.$$

(To avoid having to add these relatively large numbers, we could subtract the smallest number (5) from all 8 numbers, calculate their average and add 5 to it afterwards.)

In order to preserve the average, we must remove two numbers that have the same distance to 12. For example, if 17 would be removed then also $12 - (17 - 12) = 12 - 5 = 7$ would have to be removed. But 7 is not one of the provided numbers, so 17 is not removed.

By continuing this way we can discard all numbers, except 14 and 10. 14 is $14 - 12 = 2$ away from the average and $12 - 2 = 10$ is one of the provided numbers.

Hence, 14 and 10 can be removed without changing the average of the original eight numbers.

首先计算这 8 个数的平均值：

$$17 + 13 + 5 + 10 + 14 + 9 + 12 + 16 = 96$$

$96 / 8 = 12$ (为了避免运算相对较大的数，我们可以先将这 8 位数同时减去 5 计算出平均值后再加上 5 即为最终平均值)

要保持平均值不变，我们移除的两个数与 12 的差值必须相同。例如，如果要移除 17，则应移除 $12 - (17 - 12) = 12 - 5 = 7$ ，但是上述 8 位数中没有 7，因此不能移除 17。

重复上述方法，我们可以排除除了 14 和 10 以外的其他的数。14 与平均值的差值为： $14 - 12 = 2$ ，而 $12 - 2 = 10$ ，而且 10 在上述 8 位数之中；因此，我们可以移除 14 和 10 而保持平均值不变。

In a lottery 6 numbers are picked randomly from 1,2,3,...,48,49 each at most once. What is the probability that the 6 numbers obtained from a lottery establish an arithmetic sequence? The order they are taken does not have to be their order in the arithmetic sequence.

In the options we use the notation $n!=1\times 2\times 3\times \dots\times (n-1)\times n$ for what is called ' n factorial'.

Hint: In an arithmetic sequence the difference between any two consecutive terms is constant.

- a. $49! / (7! \times 42!)$
- b. $216 \times 6! \times 43! / 49!$
- c. $49! / (6! \times 43!)$
- d. $324 \times 43! / 49!$
- e. $6 \times 6! \times 43! / 49!$
- f. $6 \times 43! / 49!$
- g. $54 \times 6! \times 43! / 49!$

在一次抽奖活动中，从1,2,3,……, 48,49中随机抽取6个数，每个数最多被抽取一次。从抽奖中抽取的6个数字形成等差数列的概率是多少？它们被抽取的顺序不一定是它们在等差数列中的顺序。

在选项中我们使用 $n!=1\times 2\times 3\times \dots\times (n-1)\times n$ 表示“ n 的阶乘”

提示：在等差数列中，任意两个连续项之间的差值不变。

- a. $49! / (7! \times 42!)$
- b. $216 \times 6! \times 43! / 49!$
- c. $49! / (6! \times 43!)$
- d. $324 \times 43! / 49!$
- e. $6 \times 6! \times 43! / 49!$
- f. $6 \times 43! / 49!$
- g. $54 \times 6! \times 43! / 49!$

Solution

To compute this probability we first compute the number C of all combinations of 6 numbers out of 49, then the number A of arithmetic sequences of 6 numbers in the range 1..49 and finally compute the probability P of picking such a sequence as $P=A/C$.

For C we get '49 choose 6' = $49! / (6! \times (49-6)!) = 49! / (6! \times 43!)$. Justification: There are 49 options for picking the first number, for each of these there are 48 options for the second number and so on, in total $49 \times 48 \times \dots \times 44 = 49! / 43!$ options to pick 6 numbers if the order of picking them matters. These same 6 numbers can be picked in $6!$ many different orders, but for us the order of picking them does not matter. So, if these $6!$ many sequences count as one combination then we have to divide $49! / 43!$ by $6!$ to get for the number of combinations $C=49! / (6! \times 43!)$.

Let us now compute the number A of arithmetic sequences a_1, a_2, \dots, a_6 . In every such sequence two successive numbers differ by the same constant $d = a_i - a_{i-1}$. Because $49 \geq a_6 = a_1 + 5d$ this poses restrictions on d and a_1 . From $a_1 \geq 1$ follows $5d \leq 49 - 1 = 48 \rightarrow 1 \leq d \leq 9$. The following table shows in the 3rd column how many values a_1 can take for each value of d .

d	$5d$	$49 - 5d$
1	5	44
2	10	39
3	15	34
4	20	29
5	25	24
6	30	19
7	35	14
8	40	9
9	45	4

To get A we need to add up these numbers.

$$\begin{aligned} A &= 4+9+\dots+39+44 = (4+44) + (9+39) + (14+34) + (24+29) + 24 \\ &= 4 \times 48 + 24 = 4 \times (48 + 6) = 4 \times 54 = 216. \end{aligned}$$

For the probability we get

$$P = A/C = 216 \times 6! \times 43! / 49!$$

and with more time

$$P = 3^2 / (7^2 \times 11 \times 23 \times 47) = 9/582659 .$$

要计算概率首先我们需要计算出从 49 个数中抽取 6 个数可以有多少种排列组合 (C)，然后算出从 1 到 49 中任选 6 个数可以组成多少个等差数列 (A)，最后可以算出得到这样一个等差数列的概率为： $P=A/C$

“从 49 个中选择 6 个”则 $C = 49! / (6! \times (49-6)!) = 49! / (6! \times 43!)$ 理由：选择第一个数字时有 49 种选择，每一个数列选择第二个数字时则还有 48 种选择……以此类推，如果考虑顺序的话，选择 6 个数一共有 $49 \times 48 \times \dots \times 44 = 49! / 43!$ 种选择，而同样的 6 个数可以有 6! 种排列顺序，而且在本题中选择顺序并不重要，因此如果把 6! 个数列作为 1 个组合，我们得用 $49! / 43!$ 除以 6! 得到组合数 $C = 49! / (6! \times 43!)$

接下来计算等差数列 a_1, a_2, \dots, a_6 的个数，在每一个数列中，连续两项的差值相同 $d = a_i - a_{i-1}$ 。因为 $49 \geq a_6 = a_1 + 5d$ ，限定了 d 和 a_1 的取值范围，又 $a_1 \geq 1$ 可得 $5d \leq 49 - 1 = 48 \rightarrow 1 \leq d \leq 9$ 。下表第 3 列 列举了对于每一个 d 值对应的 a_1 的取值个数

d	5d	49 - 5d
1	5	44
2	10	39
3	15	34
4	20	29
5	25	24
6	30	19
7	35	14
8	40	9
9	45	4

要想得到 A 我们要把 a_1 可能的取值个数加总

$$A = 4+9+\dots+39+44 = (4+44) + (9+39) + (14+34) + (24+29) + 24 \\ = 4 \times 48 + 24 = 4 \times (48 + 6) = 4 \times 54 = 216$$

则概率为

$$P = A/C = 216 \times 6! \times 43! / 49!$$

如果有时间还可化简为

$$P = 3^2 / (7^2 \times 11 \times 23 \times 47) = 9/582659 .$$